Finding A Matrix (May 12-75

Given two vectors a and b and

(1) $\varphi \cdot a = b$

Find φ

take any convenient third vector c and write the Ans

(2) $\varphi = (bc)/a.c$

To prove it;

$\varphi \cdot a = (bc) \cdot a / a.c = b(c.a)/a.c = b.$

We give a couple or so of examples

May 12, 1975

$a = 1 + 3 + 2$

$b = 2 + 4 + 5$

$c = 1 + 1 + 1$

$$(bc) = (2+4+5)(1+1+1) = \begin{matrix} 2+2+2 \\ 4+4+4 \\ 5+5+5 \end{matrix}$$

$a \cdot c = 6$

$$\varphi = (1/6)\begin{vmatrix} 2+2+2 \\ 4+4+4 \\ 5+5+5 \end{vmatrix}$$

$$\varphi \cdot a = \frac{1}{6}\begin{vmatrix} 2+2+2 \\ 4+4+4 \\ 5+5+5 \end{vmatrix}\begin{vmatrix} 1 \\ 3 \\ 2 \end{vmatrix} = 1/6 \begin{vmatrix} 12 \\ 24 \\ 30 \end{vmatrix} = \begin{vmatrix} 2 \\ 4 \\ 5 \end{vmatrix} = b$$

Example 2

$a = 3 + 1 + 2 + 4$

$b = 1 + 4 + 3 + 5$

$c = 1 + 1 + 1 + 1$

$$(bc) = \begin{matrix} 1+1+1+1 \\ 4+4+4+4 \\ 3+3+3+3 \\ 5+5+5+5 \end{matrix} \qquad a \cdot c = 10$$

$$\varphi = (1/10)\begin{vmatrix} 1+1+1+1 \\ 4+4+4+4 \\ 3+3+3+3 \\ 5+5+5+5 \end{vmatrix}$$

May 12, 1975

$$\varphi \cdot a = (1/10)\begin{vmatrix} 1 + 1 + 1 + 1 \\ 4 + 4 + 4 + 4 \\ 3 + 3 + 3 + 3 \\ 5 + 5 + 5 + 5 \end{vmatrix}\begin{vmatrix} 3 \\ 1 \\ 2 \\ 4 \end{vmatrix} = 1/10\begin{vmatrix} 10 \\ 40 \\ 30 \\ 50 \end{vmatrix} = \begin{vmatrix} 1 \\ 4 \\ 3 \\ 5 \end{vmatrix} = b$$

Example 3

$$a = 1 - 1 + 2$$
$$b = 1 + 5$$
$$c = 1 + 1 + 1$$

$$bc = \begin{matrix} 1 + 1 + 1 \\ 5 + 5 + 5 \end{matrix}$$

$$a \cdot c = 2$$

$$\varphi = (1/2)\begin{vmatrix} 1 + 1 + 1 \\ 5 + 5 + 5 \end{vmatrix}$$

$$\varphi \cdot a = (1/2)\begin{vmatrix} 1 + 1 + 1 \\ 5 + 5 + 5 \end{vmatrix}\begin{vmatrix} 1 \\ -1 \\ 2 \end{vmatrix} = (1/2)\begin{vmatrix} 2 \\ 10 \end{vmatrix} = \begin{vmatrix} 1 \\ 5 \end{vmatrix} = b$$

The formula

$$\boxed{\varphi = (bc)/a \cdot c}$$

is true for all dimensions

$$\varphi \cdot a = b$$