Some procedures in this book call for the computation of column co-factors of matrices. A possibly more computationally efficient method of computing column co-factors is illustrated and applied to the solution of systems of equations.

Given a system of equations to be solved:

AX = B

X1	X2	X3	X4		B
1	1	1	1	=	-1
0	-1	-1	-1	=	2
5	5	5	3	=	-3
-2	-3	-4	-4	=	4

The objective is to first reduce the augmented matrix above to the form:

x	x	x	x	x
x	x	x	x	0
x	x	x	0	0
x	x	0	0	0

where x denotes any number.

The reduction is carried out in the usual elimination procedure. The following steps will result in the properly reduced matrix:

step 1: 2 * row 1 added to row 2

step 2: -3 * row 1 added to row 3

step 3: 4 * row 1 added to row 4

The reduced matrix is:

1	1	1	1	-1
2	1	1	1	0
2	2	2	0	0
2	1	0	0	0

We now form a four-component vector (GAMMA) which is orthogonal to rows 2-4 of the reduced matrix above.

Starting with row 4, we exchange the first two elements and reverse the sign of the first (or second) element:

GAMMA = (-1 2 u v)

where u,v are yet to be determined. Notice that vector GAMMA is orthogonal to row 4 of the reduced matrix:

(GAMMA) * (row 4) = 0

(-1 2 u v) * (2 1 0 0) = 0

To find the value of u, we set the dot product of vector GAMMA and row 3 of the reduced matrix equal to zero:

(GAMMA) * (row 3) = 0

(-1 2 u v) * (2 2 2 0) = 0

-2+4+2*u+0 = 0

u = -1

The vector GAMMA is now:

GAMMA = (-1 2 -1 v)

with component v yet to be determined. To find the value of v, we set the dot product of vector GAMMA and row 2 equal to zero:

(GAMMA) * (row 2) = 0

(-1 2 -1 v) * (2 1 1 1) = 0

-2+2-1+v = 0

v = 1

The vector GAMMA is therefore:

GAMMA = (-1 2 -1 1)

The components of vector GAMMA differ from the solution vector of the system of equations to within a scale factor. This scale factor can be determined by requiring that scale factor k times the dot product of vector GAMMA and row 1 of the reduced matrix equal the right hand side of row 1:

k(GAMMA * row 1) = -1

k(-1 2 -1 1) * (1 1 1 1) = -1

k = -1

Applying the scale factor k to the components of vector GAMMA gives the solution vector to the system of equations whose components are:

X1 = 1

X2 = -2

X3 = 1

X4 = -1

The components of vector GAMMA are equivalent to the column co-factors to within a scale factor. For large problems, the computation of the GAMMA vector components as illustrated in this example may be faster then computing them by the column co-factor technique.