Line Voiding Operations

In the unpublished Mutation Geometry it is shown how to put a line of zeroes thru any prechosen path thru a matrix of order n. To this end we note that if we have a vector defined by :

(1) $$a = (a_1 , a_2)$$

then

(2) $$\hat{a} = (a_2 , -a_1)$$

is a vector normal to a. One may set up a transformation and prove that $\hat{a}$ is normal to a but this would be too slow. (2) does not say that it is the only answer nor do we care. That (2) is one answer may be easily tested:

(3) $$a \cdot \hat{a} = (a_1 , a_2).(a_2 , -a_1) = a_1 a_2 - a_2 a_1 = 0$$

The components of a in (1) may be looked upon as a matrix. Then the components of $\hat{a}$ in (2) are the cofactors in order from left to right of the elements in (1). This is a generalization the power of which you will soon see.

If we have two vectors

$$a_1 = (a_{11}, a_{12}, a_{13}) \tag{4}$$

$$a_2 = (a_{21}, a_{22}, a_{23}) \tag{5}$$

Then

$$b = (b_1 , b_2 , b_3) \tag{6}$$

is a vector normal to a and a where b_1 , b_2 , b_3 are the column cofactors in order from left to right in the system

$$a_1$$

$$a_2$$

$$b_1 = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \qquad b_2 = -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \qquad b_3 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

This fact may be attested to by actual algebraic multiplication.

If one has three vectors

$$a_1 = (a_{11} , a_{12} , a_{13} , a_{14})$$

$$a_2 = (a_{21} , a_{22} , a_{23} , a_{24})$$

$$a_3 = (a_{31} , a_{32} , a_{33} , a_{34})$$

Then

$$b = (b_1 , b_2 , b_3 , b_4) \tag{7}$$

is a vector normal to a_1 a_2 a_3 , where the components of b are the column cofactors in order from left to right in the system

$$a_1$$
$$a_2$$
$$a_3$$

This may be shown by direct algebraic multiplication. In general if we have n - 1 vectors

$$a_1 = (a_{11} ,a_{12} ,a_{13} .... ,a_{1n})$$
$$a_2 = (a_{21} ,a_{22} ,a_{23} .... ,a_{2n})$$
$$..............................$$
$$a_{n-1} = (a_{n-1,1} ,a_{n-1,2} ,a_{n-1,3} .... ,a_{n-1,n})$$

Then

$$(8) \qquad b = (b_1 ,b_2 ,b_3 .... ,b_n)$$

is a vector normal to the system a_1 a_2 .... a_{n-1}. It is too long to test the truth of this by direct algebraic multiplication. For a short proof identify b with a in equation (11) and the n - 1 a in the right side of (11) with our a above. Then the scalar product of b with any one of he a in the right of (11) is identicall zero , being the volume of a hyper-parallelogram with two sides equal.

Epp-Vectors And Shadow Matrices
(SPACE VOIDING)

To any given hyper vector

$$(1) \qquad a = (a_{11}, a_{12} , a_{13} .... a_{1n})$$

one may construct an indefinite number of normal hyper vectors, one among which is:

$$(2) \qquad \hat{a} = (-a_{12} , a_{11} , 0 .... 0)$$

To any two hypervectors

$$(3) \qquad a_1 = (a_{11} , a_{12} , a_{13} \dots\dots a_{1n})$$

$$(4) \qquad a_2 = (a_{21} , a_{22} , a_{23} \dots\dots a_{2n})$$

one may construct a normal vector:

$$(5) \qquad b = (b_1 , b_2 , b_3 \dots\dots 0)$$

where the first three components of b are the column cofactors (of first three columns) in order from left to in the system (3), (4) with all the other components of b being zero. This construction may be accomplished in many ways. It is at the discretion of the operator. For instance if there were a patch of large numbers in the first two vectors I would let the components in b corresponding to them be zero and take cofactors of the remaining terms, a saving of time and labor.

To any three hyper-vectors

$$(6) \qquad a_1 = (a_{11} , a_{12} , a_{13} , a_{14} , a_{15} \dots\dots a_{1n})$$

$$(7) \qquad a_2 = (a_{21} , a_{22} , a_{23} , a_{24} , a_{25} \dots\dots a_{2n})$$

$$(8) \qquad a_3 = (a_{31} , a_{32} , a_{33} , a_{34} , a_{35} \dots\dots a_{3n})$$

one may construct a normal vector

$$(9) \qquad b = (b_1 , b_2 , b_3 , b_4 , 0 \dots\dots 0)$$

where the first four components of b are the column cofactors (of the first 4 columns) in order from left to right , all the other components in b being zero. In general one may construct a vector normal to n-1 hyper-vectors with n components, there being only one way for the b. This construction possibility, as we shall see, is a powerful tool in analysis and system solutions.

Any vector formed fron a system of vectors, as the b above, we call an epp-vector. Any n by n-1 matrix formed from epp-vectors we call a shadow matrix.

Consider the system of equations:

$$(10) \qquad \begin{array}{ccccc|c|c} a_{11} & a_{12} & a_{13} & a_{14} \dots\dots & a_{1n} & x_1 & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} \dots\dots & a_{2n} & x_2 & b_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} \dots\dots & a_{nn} & x_n & b_n \end{array} \quad = $$

We now form the shadow matrix:

$$
(11) \qquad \begin{matrix} c_{11} & c_{12} & c_{13} & cc_{14} \cdots & c_{1n} \\ c_{21} & c_{22} & c_{23} & c_{24} \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{n-1,1} & c_{n-1,2} & c_{n-1,3} & c_{n-1,4} \cdots & c_{n-1,n} \end{matrix}
$$

where the first row is the epp-vector formed orthogonally to the n-1 column vectors to the right of the first column in (10). The second row is the epp-vector formed orthogonally to the n-2 columns to the right of the first 2 columns in (10). The jth row is the epp-vector formed orthogonally to the n-j column vectors to the right of the first j columns in (10). j going from 1 to n-1; the last row being the epp-vector formed orthogonally to the last column vector in (10). It is the n-1 th row.

We now multiply both sides of (10) by (11) and get:

$$
(12) \qquad \begin{matrix} d_{11} & 0 & 0 & 0 & 0 & \cdots & 0 \\ d_{21} & d_{22} & 0 & 0 & 0 & \cdots & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ d_{n-1,1} & d_{n-1,2} & d_{n-1,3} & d_{n-1,4} & d_{n-1,5} & \cdots & 0 \end{matrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ \cdots \\ x_4 \\ x_n \end{vmatrix} = \begin{vmatrix} e_1 \\ e_2 \\ e_3 \\ \cdots \\ e_{n-1} \end{vmatrix}
$$

Thus triangleising the system. From (12) we have for x the expression:

$$
(13) \qquad x = e_1 / d_{11}
$$

and the values for the other x follow immediately. We do an illustrative examlpe. Consider the system;

$$
\begin{vmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 2 & 3 & -2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 7 \\ 5 \\ 1 \end{vmatrix}
$$

One shadow matrix is

(15)
$$\begin{matrix} -8 & 7 & 3 \\ 2 & -1 & 0 \end{matrix}$$

Multiplying both sides of (14) by (15) we get:

(16)
$$\begin{vmatrix} -9 & 0 & 0 \\ 3 & 3 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} \quad \begin{vmatrix} -18 \\ 9 \end{vmatrix}$$

(17)
$$x_1 = -18/-9 = 2$$
$$x_2 = (9 - 6)/3 = 1$$
$$x = 3$$

In this case three matrices of shadow form may be formed. They all lead to the same answers. We do a second illustrative example. Consider the system.

(1)
$$\begin{vmatrix} 2 & -2 & 2 & 1 \\ 1 & 2 & -1 & 2 \\ -1 & -1 & -2 & -3 \\ 3 & -1 & -1 & -1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} = \begin{vmatrix} -1 \\ 2 \\ -2 \\ 1 \end{vmatrix}$$

A shadow matrix for (1) is:

(2)
$$\begin{matrix} 6 & 5 & 9 & -11 \\ 7 & 4 & 5 & 0 \\ 2 & -1 & 0 & 0 \end{matrix}$$

Multiply both sides of (1) by (2) and we get;

$$(3) \qquad \begin{vmatrix} -25 & 0 & 0 & 0 \\ 13 & -11 & 0 & 0 \\ 3 & -6 & 5 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} = \begin{vmatrix} -25 \\ -9 \\ -4 \end{vmatrix}$$

$$(4) \qquad \begin{aligned} x_1 &= -25/-25 = 1 \\ x_2 &= (-9 -13)/-11 = 2 \\ x_3 &= (-4 -3 + 12)/5 = 1 \\ x_4 &= -1 \end{aligned}$$

This is almost on a par with the first scheme of solution. This same process may be used to diagonalize a matrix. It is about twice the work of triangleation and so we would not recommend it . However, for completeness we do an illustrative example or two: Solve the system

$$(5) \qquad \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & 2 \\ 3 & -1 & -1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 8 \\ 6 \\ 2 \end{vmatrix}$$

by diagonalation. Construct a shadow matrix of three rows, normal respectively to the second and third, first and third, and first and second columns of (5) respectively. The shadow matrix is:

$$(6) \qquad \begin{vmatrix} 1 & 0 & 1 \\ -7 & 5 & 3 \\ 1 & 1 & -1 \end{vmatrix}$$

Multiplying both sides of (5) by (6) one obtains

$$(7) \quad \begin{vmatrix} 5 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & 4 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 10 \\ -20 \\ 12 \end{vmatrix}$$

$$x_1 = 10/5 = 2$$
$$x_2 = -20/-20 = 1$$
$$x_3 = 12/4 = 3$$

Solve the system

$$(8) \quad \begin{vmatrix} 2 & -2 & 2 & 1 \\ 1 & 2 & -1 & 2 \\ -1 & -1 & -2 & -3 \\ 3 & -1 & -1 & -1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} = \begin{vmatrix} -1 \\ 2 \\ -2 \\ 1 \end{vmatrix}$$

by diagonalàtion. Construct a shadow matrix of four rows normal respectively to the (2,3,4), (3,4,1), (4,1,2) and (1,2,3) columns of (8) . The shadow matrix is

$$(9) \quad \begin{vmatrix} 6 & 5 & 9 & -11 \\ 23 & 15 & 22 & -13 \\ -14 & -20 & -21 & 9 \\ -19 & -20 & -16 & 14 \end{vmatrix}$$

Multiply both sides of (8) by (9) and one obtains

$$\begin{vmatrix} -25 & 0 & 0 & 0 \\ 0 & -25 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & -25 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} = \begin{vmatrix} -25 \\ -50 \\ 25 \\ 25 \end{vmatrix}$$

$$x_1 = -25/-25 = 1 , \quad x_2 = -50/-25 = 2$$
$$x_3 = 25/25 = 1 , \quad x_4 = 25/-25 = -1$$