Matrix inversion

Given a matrix

$$A = \begin{matrix} a_{11} & a_{12} & a_{13} & \cdot\cdot & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & ,,, & a_{nn} \end{matrix}$$

We write : R (a, b, c, d, .. n)

where a, b, c, ... n are the first, second, third, .. n th columns of A. Then

g^{-a}, g^{-b}, ... g^{-n}

are perpendicular to all the columns of which it is not a member. Then

$g^{-a}/\ a.g^{-a}$ is a unitary vector and in like mannar for the

other vectors. Thus the matrix with rows :

$$\begin{matrix} g^{-a}/\ a.g^{-a} \\ g^{-b}/\ b.g^{-b} \\ g^{-c}/\ c.g^{-c} \\ \cdots\cdots \\ g^{-n}/\ n.g^{-n} \end{matrix} = A^{-1}$$

is the inverse of matrix A. We do an example:

$$A = \begin{matrix} 2 & 3 & 1 & 4 \\ 1 & -2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ -1 & 1 & -2 & -1 \end{matrix}$$

a	=	2	1	3	-1
b	=	3	-2	-1	2
c	=	1	-1	2	-2
d	=	4	3	1	-1

g^{-a}	=	0	0	1	1
g^{-b}	=	1	-1	0	1
g^{-c}	=	1	-7	-6	-23
g^{-d}	=	5	7	-9	-10

$a.g^{-a} = 2$, $b.g^{-b} = 6$, $c.g^{-c} = 42$, $d.g^{-d} = 42$ then

$$A^{-1} = \begin{pmatrix} (\ 0 & 0 & 1 & 1\ )/2 \\ (\ 1 & -1 & 0 & 1\ )/6 \\ (\ 1 & -7 & -6 & -23\ )/42 \\ (\ 5 & 7 & -9 & -10\ )/42 \end{pmatrix}$$

By actual multiplication one gets:

$$A.\,A^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = A^{-1}.\,A$$

Usually the inverse is written with a common denominator, the determinant of the matrix, but this is not necessary.The determinant of the matrix above is 84. To put ours into the usual shape one could multiply by 84/84.

We point out again that the amount of work in computing g^{-a}, g^{-b}, g^{-c}, ... g^{-n} diminishes as one goes till the last one g^{-n} only requires back substitution for its c computation. This is a great saving of labor. Observe it.