A New Mode of Inversion of Matrices

For simplicity we start with a third order matrix:

$$A = \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

We first find vectors

$$C = \begin{matrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{matrix}$$

which, in tern, are perpendicular to columns (2 and 3), (1 and 3) and (1 and 2) . We multiply A and C and get the diagonal matrix

$$\begin{matrix} M_{11} & 0 & 0 \\ 0 & M_{22} & 0 \\ 0 & 0 & M_{33} \end{matrix}$$

In general the M may not be equal nor have the same sign. WE want the M to be equal and have a positive sign. Let M be the least common multiple of the M_{ij} then we can write:

$M = d_1 M_{11} = d_2 M_{22} = d_3 M_{33}$. We can now write the inverse of A:

$$A^{-1} = \left. \begin{matrix} d_1 (& c_{11} & c_{12} & c_{13}) \\ d_2 (& c_{21} & c_{22} & c_{23}) \\ d_3 (& c_{31} & c_{32} & c_{33}) \end{matrix} \right/ M$$

We now do a numerical of the 4 th order:

$$A = \begin{matrix} 2 + 1 - 1 + 3 \\ 1 + 2 + 1 - 2 \\ 3 - 2 + 2 + 1 \\ - 2 - 1 + 3 - 1 \end{matrix}$$

$$C = \begin{matrix} 5 - 14 - 22 + 21 \\ 15 + 14 - 10 + 7 \\ - 19 - 14 - 6 - 35 \\ - 37 + 14 + 6 - 21 \end{matrix}$$

$$C \cdot A = \begin{matrix} 112 + 0 + 0 + 0 \\ 0 + 56 + 0 + 0 \\ 0 + 0 - 112 + 0 \\ 0 \quad 0 \quad 0 - 112 \end{matrix}$$

$$M = 1 (112) = 2 (56) = - 1 (- 112) = - 1 (- 112)$$

$$d_1 = 1, \qquad d_2 = 2, \qquad d_3 = - 1, \qquad d_4 = - 1$$

Multiplying the c rows by the d_i and divide by M we get the inverse of A:

$$A^{-1} = \begin{matrix} 5 - 14 - 22 + 21 \\ 30 + 28 - 20 + 14 \\ 19 + 14 + 6 + 35 \\ 37 - 14 - 6 + 21 \end{matrix} \Bigg/ 112$$

It is an easy extension to the general case. I do one of the fith order just to help you see the generalization which one must be prepared to do.

$$A = \begin{array}{ccccc} 2 & -1 & -1 & +3 & +1 \\ 1 & +2 & +1 & -2 & +2 \\ 3 & -2 & +2 & +1 & -1 \\ -2 & -1 & +3 & -1 & -3 \\ -1 & -1 & +2 & +1 & -2 \end{array}$$

$$C = \begin{array}{ccccc} 25 & -6 & +18 & +41 & -64 \\ -25 & -2 & +6 & -33 & +32 \\ 15 & -26 & -2 & +31 & -64 \\ 5 & +2 & -6 & -27 & +48 \\ -25 & +14 & -2 & -49 & +56 \end{array}$$

$C \cdot A = 80 - 40 - 80 + 80 + 40$

$M = 1\,(80) = -2\,(-40) = -1\,(-80) = 1\,(80) = 2\,(40)$

$d_1 = 1, \quad d_2 = -2. \quad d_3 = -1, \quad d_4 = 1, \quad d_5 = 2$

$$A^{-1} = \begin{array}{ccccc} 25 & -16 & +18 & +41 & -64 \\ 50 & +4 & -12 & +66 & -64 \\ -15 & +26 & +32 & -31 & +64 \\ 5 & +2 & -6 & -27 & +48 \\ -50 & +28 & -4 & -98 & +112 \end{array} \Big/ 80$$

This answer checks: $A \cdot A^{-1} = A^{-1} \cdot A = I$

I did one of the 10 th order which is found fully illustrated on pages 217 ... in vol. 6. The whole scheme is a matter of efficiently calculating the various γ vectors forming the C matrix. Most of the work is in calculating the first γ The other γ have so many parts alike the first γ that one has mostly to copy them down.