Inversion of A Matrix using Arbitrary Elements.

Given:

A = to a square matrix of $n \times n$ dimension.

Arbitrarily choose any n vectors $R_1, R_2, R_3 \ldots R_n$. Then one can obtain

$$
(1) \quad
\begin{array}{lll}
A \cdot R_1 = S_1 & & R_1 = A^{-1} \cdot S_1 \\
A \cdot R_2 = S_2 & & R_2 = A^{-1} \cdot S_2 \\
A \cdot R_3 = S_3 & \text{or} & R_3 = A^{-1} \cdot S_3 \\
- - - - - & & - - - - - \\
A \cdot R_n = S_n & & R_n = A^{-1} \cdot S_n
\end{array}
$$

Then one sees that:

$$(2) \quad A^{-1} = R_1 S_1^{-1} + R_2 S_2^{-1} + R_3 S_3^{-1} + \cdots R_n S_n^{-1}$$

Since it satisfies the system above. Here S_x^{-1} is the reciprocal of S_x with respect to the other S of the system. In other words

(3) $S_k^{-1} \cdot S_k = 1$ and $S_h^{-1} \cdot S_h = 0$ for all $h \neq k$. S_k^{-1} is found from the column cofactors of the other S es.

If one has a system of equations given by

(4) $A \cdot r = s$

then $r = A^{-1} s =$

(5) $r = R_1 (S_1^{-1} \cdot s) + R_2 (S_2^{-1} \cdot s) + \cdots R_n (S_n^{-1} \cdot s)$

= the answer

Equation (2) has many intriguing possibilities since the various R are arbitrary and can be experimented with. One possibility is to calculate the various R so that the resulting S are self-reciprocal. In other words $S_h^{-1} = S_h$.

We shall do a numerical illustrative example or two

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We point out here, as we have done so often, that the inversion of a matrix is NOT a good way to solve a system of equations. One may need the inverse of a matrix for other purposes.