Chapter 3.

1 The Meaning of γ Vectors.

In high school geometry it is shown that the intersection of two planes is a straight line. The equations of the two lines may be written:

$$(1) \quad a_1 x_1 + a_2 x_2 + a_3 x_3 = c_1$$

$$(2) \quad b_1 x_1 + b_2 x_2 + b_3 x_3 = c_2$$

The normals to each of these planes may be written:

$$(3) \quad N_1 = a_1 + a_2 + a_3$$

$$(4) \quad N_2 = b_1 + b_2 + b_3$$

Taking the column cofactors in order from left to right of (3) and (4) we get:

$$(5) \quad \gamma = \gamma_1 + \gamma_2 + \gamma_3$$

$$\gamma_1 = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \quad \gamma_2 = - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \quad \gamma_3 = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

We now show, by actual multiplication, that γ is perpendicular to each of the normals N_1 and N_2.

$$\gamma \cdot N_1 = a_1 \begin{matrix} a_2 & a_3 \\ b_2 & b_3 \end{matrix} - a_2 \begin{matrix} a_1 & a_3 \\ b_1 & b_3 \end{matrix} + a_3 \begin{matrix} a_1 & a_2 \\ b_1 & b_2 \end{matrix}$$

$$= a_1 (a_2 b_3 - a_3 b_2) - a_2 (a_1 b_3 - a_3 b_1) + a_3 (a_1 b_2 - a_2 b_1)$$

$$= a_1 a_2 b_3 - a_1 a_3 b_2 - a_2 a_1 b_3 + a_2 a_3 b_1 + a_3 a_1 b_2 - a_3 a_2 b_1 = 0$$

and in the same way:

$$\gamma \cdot N_2 = 0.$$

Thus the γ so constructed is perpendicular to each of the normals to the two planes and thus has the direction of the line of intersection of the two planes.

Note here that 2 planes in a 3 dimensional space determine a line.

We can show in general that $n - 1$ planes in an n dimensional space determine a line (a γ vector). This γ vector has the direction of the line of intersection of the $n - 1$ planes. This is a CARDINAL PRINCIPLE of Mutation Geometry for linear programming.

Suppose we have $n - 1$ hyper-planes in an n dimension space. We may write them as:

$$(6) \quad \begin{matrix} a_{11} x_1 + a_{12} x_2 + \dots a_{1n} x_n = b_1 \\ a_{21} x_1 + a_{22} x_2 + \dots a_{2n} x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{j1} x_1 + a_{j2} x_2 + \dots a_{jn} x_n = b_j \\ j = n - 1 \end{matrix}$$

The normals to each of these is:

$$(7) \quad \begin{matrix} N_1 = a_{11} + a_{12} + \dots a_{1n} \\ N_2 = a_{21} + a_{22} + \dots a_{2n} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ N_j = a_{j1} + a_{j2} + \dots a_{jn} \end{matrix}$$

Taking column cofactors in order from left to right in (7) we get:

$$\gamma = M_1 + M_2 + M_3 + \ldots M_n$$

We now take products of with each of the normals N_1 and get:

$$\gamma \cdot N_1 = 0$$

$$\gamma \cdot N_2 = 0$$

$$\gamma \cdot N_j = 0$$

since each product is the value of a determinant with two of its rows the same.

Thus we have proven that our γ vector is perpendicular to each of the normals of the n - 1 hyper-planes and thus has has the direction of their line of intersection.

In the case of 2 planes in 3 dimensions each plane passes thru their line of intersection. A number of planes passing thru a common line is called a sheaf of planes and their common line the axis of the sheaf.

Our gamma vector γ is the axis of a sheaf of planes and this sheaf is always associated with it. They give more gammas when the gamma strikes other planes (splattered gammas). These other planes, in linear programming, will be the faces of the polyhedrons formed by the systems of equations of constraint, rather inequalities of constraint.

We use the column cofactor scheme of calculating the gammas for theoretical purposes only. They are to cumbersome for practical calculations of the gammas. Later on we shall also illustrate a relatively easy scheme for the inversion of matrices by means of the gamma vectors. Gamma vectors are a construct of Mutation Geometry, a truly Pan-geometry, unifying the geometric field. High school, College, Analytic, and Projective Geometry are special cases of the New Science of Mutation Geometry.