March 25, 1983

Hull-Transversal Solution

We state the following linear programming problem;

Maximize the objective function;

$$P = C * R$$
$$= (12 \quad 1 \quad 1) * R$$

subject to:

(4) $(1 \quad 4 \quad -3) * R < 10$

(5) $(5 \quad 6 \quad -8) * R < 15$

(6) $(1 \quad -3 \quad 4) * R < 10$

(7) $(1 \quad 1 \quad -1) * R < 4$

We leave the point (3 0 0) which has the index (5, 2, 3). Any line leaving the point (3 0 0) must make an obtuse angle with the normals of each hyperplane whose equation occurs in the index of the point if the line is to stay inside the polyhedron of constraints . Thus we may write the equation of any such line as:

(8) $R = (3 \quad 0 \quad 0) + t * (2 \quad 1 \quad 3)$

This vector (2 1 3) was chosen semi-arbitrarily.

Note that

$$(2 \quad 1 \quad 3) * (0 \quad -1 \quad 0) < 0$$
$$(2 \quad 1 \quad 3) * (0 \quad 0 \quad -1) < 0$$
$$(2 \quad 1 \quad 3) * (5 \quad 6 \quad -8) < 0$$

which means that our equation (8)

(8) $R = (3 \quad 0 \quad 0) + t * (2 \quad 1 \quad 3)$

moves out through the polyhedron and not along its surface.

No calculations were done for (2 1 3).

Putting (8) into (4), (6), (7) we get:

t_6 = 7/11 for smallest t value.

Put this value of t_6 into (8) we get

R_6 = (47 7 21)/11

Thus R_6 feasibly satisfies all the constraint equations as it should. This simply means that R_6 is crossing the feasibility hull, or polyhedron of constraint. One may test for location when we are across:

$$P = C * R_6$$
$$= (12 \quad 1 \quad 1) * (47 \quad 7 \quad 21)/11$$
$$= 54$$

Appropriately, We note that R_6 hit in the face of (6). If I were dealing with a large hull one might want to go again across the hull leaving the face of (6) as we did that of (5) in the beginning of this process. I think the value of P for a point in the face of the polyhedron is less than that for a vertex in that plane. It may not be so and we can investigate this later. No matter where we finally stop on the other side of the gap we shall have to reestablish contact with a refinement process. For this purpose we shall assume that (6) is the end face.

We repeat (6) here

(6) (1 -3 4) * R < 10

The point R_6 is a point in the face of (6). We want to move in its face to some other place. Any line in the plane of (6) is perpendicular to its normal (1 -3 4). We can write these normals to (1 -3 4). They are:

(3 1 0)

(4 0 -1)

(0 4 +3)

We will take the first and write

(9) $R = R_6 + t * (3\ \ 1\ \ 0)$

Put this into (4), (5), (7) and get $t_5 = 56/21$ (minimum)

Put this into R and get;

$R_{56} = ((47\ \ 7\ \ 21) + 56/21(3\ \ 1\ \ 0))/11$

$= (1155\ \ 203\ \ 1441)/231$

therefore

$P_{56} = C * R_{56}$

$= (12\ \ 1\ \ 1) * (1155\ \ 203\ \ 1441)/231$

$= 62.8$

R_{56} is a point of the intersection of planes of (5) an (6) which is gamma(56) or;

gamma(56) = (5 6 -8)

= (0 28 21)

= (1 -3 4)

We may now write our equation of this line as

$R = R_{56} + t * (0\ \ 28\ \ 21)/231$

Put this into (4) and (7) and get:

$t_7 = 1$

therefore;

$R = R_{56} + 1 * (0\ \ 28\ \ 21)/231$

$= (1155\ \ 231\ \ 462)/231$

$= (5\ \ 1\ \ 2)$ (Index = 5 , 6 , 7)

and therefore;

$P = C * R$

= (12 1 1) * (5 1 2)

= 63 (Maximum)

It is easy to prove that this is the Maximum since we have its index and thus its neighbors.

We have gone to a lot of trouble to show in detail how this simple problem can be solved in an alternative method. It is not made for small problems. The scheme can be refined. One needs to do a lot of experiment with it to explain its possibilities. I think it will turn out that one will not have to do much more computation for a large problem than for a small one. The main computations are when one reaches the far side of the gulf where he has to re-establish the gamma connection. That has to be done whether the problem is large or small.