MUTATION GEOMETRY

CHAPTER I

Fundamental Ideas

1-1 Products

In Mutation Geometry the problems of the geometric world are divided into two categories: the alpha category and the omega category. The problems in the alpha category are associated with the alpha products: a • b, c • d, e • f, ... which are single products. Each of the two quantities in each of these alpha products is a directed quantity. a • b, for example, is to be interpreted as the ordinary product of a and the projection of b upon a or the ordinary product of b and the projection of a upon b. All alpha products are to be so interpreted. See the sketch below.

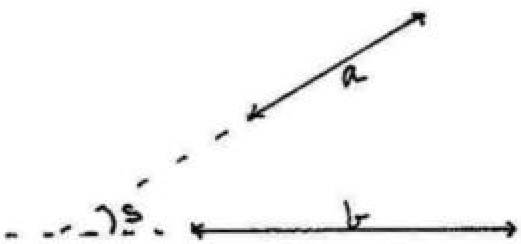

Fig. 1-1

(1) $$a \cdot b = a\, b \cos s$$

The omega products are products of the type: a • b c • d, e • f g • h .... They are double products of single alpha products. The problems in the omega category are associated with the omega type products.

In Mutation Geometry there is only one proposition. It is called omega . It is a proposition of dissolution. It splinters an omega product into the sum of two alpha porducts.

In Mutation Geometry there is only one postulate. It is called alpha. It is the statement of an enabling act. Its statement is: The alpha and omega products are required to be tempo-locally invariant. This is interpreted to mean that they may be (mentally) shifted from one local to another at any time without altering their value.

This is a fundamental principle in Mutation Geometry.

An equational statement of the omega proposition, which will be made plausible later, is:

$$(2) \qquad a{\cdot}r\, b{\cdot}r = r^2(a{\cdot}b + b_0 a \cdot \gamma)/2$$

$$(3) \qquad \gamma \wedge br$$

Equa tion (3) is to be read: gamma (γ) is the unit symmetric of b with respect to r. b_0 in (2) is the magnitude of b. The magnitudes of all quantities will be written with a zero subscript to the right. If r in (2) is unity then equation (2) takes the very useful form:

$$(4) \qquad a{\cdot}r\, b{\cdot}r = (a{\cdot}b + b_0 a{\cdot}\gamma)/2$$

If by any means one could find the sense of γ in (4) then the sense of r could be found by bisecting the angle between b and γ. See the sketch below.

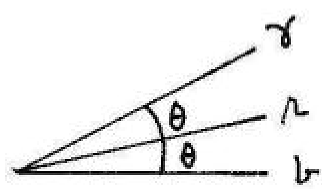

Fig. 1-2

The expression in (3) is called a mutation diagram. If we have a sum of omega products as:

$$(5) \qquad S = a{\cdot}r\, b{\cdot}r + c{\cdot}r\, d{\cdot}r + e{\cdot}r\, f{\cdot}r + \ldots$$

we may apply our omega proposition to each member and get:

$$(6) \qquad S = (a{\cdot}b + b_0 a{\cdot}\gamma_1)/2 + (c{\cdot}d + d_0 c{\cdot}\gamma_2)/2 + (e{\cdot}f + f_0 e{\cdot}\gamma_3)/2 - \ldots\ldots$$

where our mutation diagram is now represented by:

$$(7) \qquad \uparrow \left| \begin{array}{l} \gamma_1 \wedge b \quad r \\ \gamma_2 \wedge d \quad r \\ \gamma_3 \wedge f \quad r \\ \cdots\cdots \end{array} \right| \downarrow$$

Fig. 1-3

The directions of the arrows in (7) are indicative of the alpha postulate. γ_1 is the symmetric of b with respect to r. γ_2 is the symmetric of d with respect to r. γ_3 is the symmetric of f with respect to r.

Hereafter the γ_1, γ_2, γ_3 will be called transmutes; the b, d, and f .... associates.

We now show that the angle between any two transmutes is the same as the angle between their corresponding associates. See the sketch below.

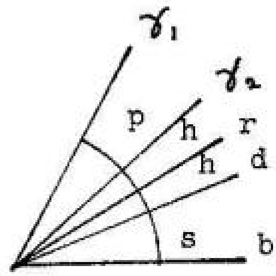

Fig. 1-4

We take any two transmutes a s γ_1 and γ_2 and their corresponding associates b and d. Let p be the angle between the transmutes γ_1 and γ_2 and s the angle between b and d. Let h be the angle between d and r and r and γ_2, these angles being equal by the definition of a symmetric. The angles between b and r and r and γ_1 are equal from the definition of a symmetric. From this last equality we may write:

$$(8) \qquad \angle (p + h) = \angle (s + h)$$

$$(9) \qquad \angle p = \angle s \qquad \text{QED}$$

This last result in (9) is of immense importance in Mutation Geometry. From this result (in harmony with the alpha postulate) we may now write our equation (6) as

(10) $$S = M + N \cdot \gamma_1$$

(11) $$M = (a \cdot b + c \cdot d + e \cdot f)/2$$

(12) $$N = ((b_{\circ}\widehat{a} + d_{\circ}\widehat{c} + f_{\circ}\widehat{e})/2$$

The quantities $\widehat{a}$, $\widehat{c}$, $\widehat{e}$, are called co-migrates. Each transmute hangs with its associate during any mental shifting of products in harmony with the alpha postulate. One may locate them by means of the mutation diagram as we shall show later. We illustrate further. We take a product such as

$$h \cdot \gamma_n$$

This may be written:

$$\widehat{h} \cdot \gamma_1$$

if $\widehat{h}$ makes the same angle with h as γ_n makes with γ_1. This is in harmony with the alpha postulate. $h \cdot \gamma_n$ and $\widehat{h} \cdot \gamma_1$ will have the same value. They are tempo-locally invariant. See the sketch below.

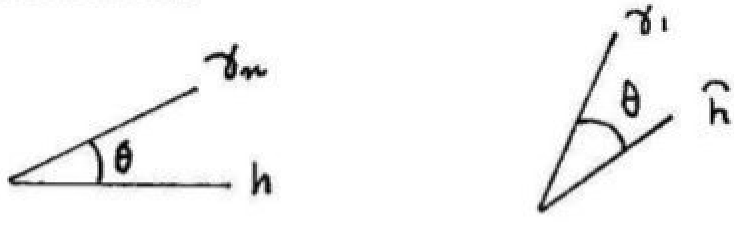

Fig. 1-5

We note that in (10) there is only a single gamma γ and that the equation (6) from which it came has several of them.

All the γ in (6) have migrated into a single γ, the choice as to which γ being at the discretion of the operator (chooser) their partners a, c, e, etc migrating at the same time into the co-migrates $\overset{\frown}{a}$, $\overset{\frown}{c}$, $\overset{\frown}{e}$, .

The composite alpha prototype , as $N \cdot \gamma_1$ in (10), into which all the γ migrate is called a primordial prototype . If we really want to be realistically symbolic about our language we may say that the mind herds the various γ into the primordial prototype or simply that we gather up the γ into a single prototype. Take your pick or still better coin your own expression for the notion.

One should realize that the alpha postulate is an elementary notion. In truth, one may find its half cousin in high school geometry: a figure may be moved from one place to another without changing its shape, nothing being said about the time required to make the change etc.

Since we have arbitrarily chosen γ_1 to remain fixed in the mental mutation transformation into the primordial prototype it is obvious that

$$\overset{\frown}{a} = a$$

This last equation means that a did nothing but what his partner γ_1 did. The Mutation Diagram pinpoints each of them for us.

To pin point the co-migrates $\overset{\frown}{c}$ and $\overset{\frown}{e}$ we go to the Mutation Diagram.

The arrow on the left points in the direction from γ_n to

γ_1 and the arrow on the right points in the same direction if we consider the left hand side and right hand sides as parts of the same circle. The direction of the arrows is clockwise. To find $\hat{c}$ draw a line equal in magnitude to c on the same side of c that d is on b and making the angle between $\hat{c}$ and c equal to the angle between b and d. This gives us $\hat{c}$. This is so since we showed that the angle between any two transmutes is the same as the angle between their corresponding associates. In the same way we find $\hat{e}$. Draw a line equal to e in magnitude on the same side of e that f is on b and making the angle between $\hat{e}$ and e equal to the angle between b and f. This line is $\hat{e}$. From this it appears that none of the stampeding co-migrates went astray, and it is FOR SURE that the field of geometry has come to " life " in truth in Mutation Geometry.

Our Mutation Diagram may have many transmutes in it but by the scheme above one can always locate the corresponding co-migrates and thus arrive at a single composite primordial prototype. Later, for completeness, we shall derive an analytic or synthetic expression for the co-migrates, but who wants to be tedious when it can be easily done mentally.

1-2 Solution for a Prototype

If by some means we can find an expression for the r (its magnitude being unity) in an alpha type product as a.r we shall have gone a long way in the New Science of Mutation Geometry. One would, indeed, have to be dull not to see this .

If all the problems of the geometric world are represented by either the alpha or omega category and if we can solve those in the alpha category and our omega proposition reduces those in the omega category to those in the alpha category then, at least, in principle we can solve the problems of the geometric world. Given tha t:

(1) $$a \cdot r = b$$

Then

(2) $$r = (b\, a \pm \check{a} \sqrt{a^2 - b^2})\, a^{-2}$$

is a solution to (1). Here r is specified to be unity. a is a directed quantity and b is a scalar. $\check{a}$ is the iso-orthogonal to a. It is equal to a in magnitude and perpendicular to a and pointing in a counterclockwise direction to a. If we put (2) into (1) we see that it satisfies. If we square r in (2) we get one.

We may further show that (2) is the only solution to (1). To show this suppose that there is an extra term in (2) . call it p. We then write for (2):

$$(3) \qquad r = (b a \pm \breve{a} \sqrt{a^2 - b^2}) \bar{a}^{2} + p$$

Solving (3) for p and squaring we get:

$$(4) \qquad p^2 = (r - (b a \pm \breve{a} \sqrt{a^2 - b^2}) \bar{a}^{-1})^2$$

$$= 1 - 2 (b a \cdot r \pm \breve{a} \cdot r \sqrt{a^2 - b^2}) \bar{a}^{-2} + 1$$

$$= 2 - 2 (b^2 + a^2 - b^2) a^{-2}$$

$$= 2 - 2 = 0$$

We thus may use (2) with perfect assurance for we have shown that it is the only solution. We have proven it only for solutions of the form of (2). There may be other solutions of a form different from that given and they may be elegant and have many beautiful properties but if they exist we are not interested in them right now. We only need one solution for (1) and equation (2) is it.

We point out here that (2) will play quite a role in triple in analytic, college, and projective geometry. We shall try to show that the New Science of Mutation Geometry is truly a Pan Geometry.

It will be expedient hereafter to call our directed quantities vectors. They need no special symbols or type for the mode of operating with them identifies them at once . One thus frees the equations from a lot of symbolic baggage.

If r is a vector then r_o will represent its magnitude and r' a unit vector in the direction of r. $\breve{r}$ will represent its iso-orthogonal. We now write a vector equation that is very useful:

(5) $$r = r_o r'$$

This holds for any vector. For the velocity of a particle we may write:

(6) $$v = v_o v'$$

The time rate of change of a vector r is written $\dot{r}$. We shall not digress here to exploit the physical possibilities inherent in (5) and (6) but merely point out in passing that they are the framework of particle dynamics.

1-3 Equation of a Straight Line

If a is the perpendicular vector distance from an origin O to a given straight line and r is any vector from O to any point on this line we may write for its equation:

(1) $$a' \cdot r = a_o$$

See the sketch below.

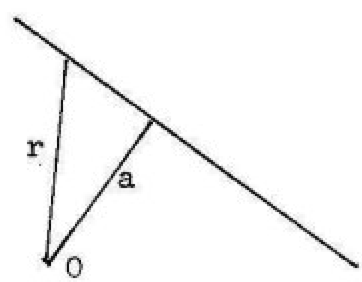

Fig. 1-6

For a pair of lines we may write:

(2) $$a' \cdot r = a_o$$

(3) $$b' \cdot r = b_o$$

We point out that the equation of a line is an alpha type product.

1-4 Common Point of two Lines

Equations (2) and (3) of paragraph 1-3 may be written:

(1) $$b_0 a' \cdot r = a_0 b_0$$

(2) $$a_0 b' \cdot r = a_0 b_0$$

Subtracting these two equations and cancelling r_0 from the resulting equation we get:

(3) $$(b_0 a' - a_0 b') \cdot r' = 0.$$

This may be written:

(4) $$A \cdot r' = 0$$

(5) $$A = (b_0 a' - a_0 b')$$

(6) $$\breve{A} = (b_0 \breve{a}' - a_0 \breve{b}')$$

Equation (4) is an alpha type equation and its solution according to (2) paragraph 1-2 is:

(7) $$r' = \pm \breve{A}/A_0 = \pm \breve{A}'$$

whence

(8) $$a' \cdot r' = \pm a' \cdot \breve{A}/A_0 = \pm a' \cdot (b_0 \breve{a}' - a_0 \breve{b}')/A_0$$

$$= \mp a_0 a' \cdot \breve{b}'/A_0$$

From (1), 1-3 we get:

(9) $$r_0 = a_0 / a'\cdot r' = \mp A_0 / a'\cdot \overset{\downarrow}{b} .$$

Now

(10) $$r = r_0 r'$$

Put (7) and (9) into (10) and we get:

(11) $$r = - (b_0 \overset{\downarrow}{a} - a_0 \overset{\downarrow}{b}) / a' \cdot \overset{\downarrow}{b}$$

This equation (11) is a very important one. We point out here that we shall be able to use it in many ways. In the theory of the conics, for instance, we shall use it to find the focus, vertex, and perfolatum of a parabola from its Cartesian equation. We shall read more into it as we go and find uses for it.

1-5 Cartesian Axes

Cartesian axes are two perpendicular lines, horizontal and vertical meeting at a point called the origin designated by O. The horizontal line is called the x axis and the vertical line is called the y axis. See the sketch below.

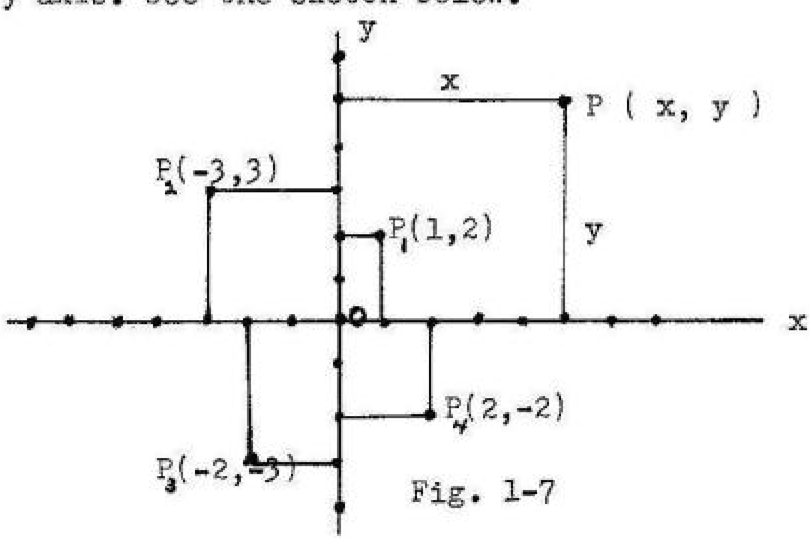

Fig. 1-7

We shall be primarily interested in points in the plane of the axes for the present . We shall designate the unit vectors along the postive x and y axes by i and j respectively. This is standard notation. The perpendicular distances from the axes to a point are called the coordinates of the point. A point is properly designated by P (x, y). The x coordinate is always written first by convention. For points in the first quadrant both x and y will be postive. For points in the second quadrant x will be negative and y postive. For points in the third quadrant the x will be negative and likewise y. For points in the fourth quadra nt x will be postive and y negative. The five points P (x, y), P (1, 2), P (-3, 3), P (-2, -3), and P (2, - 2) on the sketch are properly labeled.

The vector from the origin O to a point P (x, y) will be designated by r. It may be written:

$$r = r(x, y) = x\,i + y\,j \tag{1}$$

Other vectors will be written:

$$a = a(a_1, a_2) = a_1 i + a_2 j \tag{2}$$

$$b = b(b_1, b_2) = b_1 i + b_2 j \tag{3}$$

$$c = c(c_1, c_2) = c_1 i + c_2 j \tag{4}$$

The a_1, a_2 b_1, b_2 c_1, c_2 are the components of the vectors a, b, and c respectively. For vectors of three dimensions we write:

$$r = x\,i + y\,j + z\,k \tag{5}$$

$$a = a_1 i + a_2 j + a_3 k \tag{6}$$

For n dimensional vectors we write:

$$r = x_1 i_1 + x_2 i_2 + x_3 i_3 \;\ldots\ldots\; x_n i_n \tag{7}$$

$$a = a_1 i_1 + a_2 i_2 + a_3 i_3 \;\ldots\ldots\; a_n i_n \tag{8}$$

Here k is a unit vector along the z axis which is an axis perpendicular to both the x and y axes. For n dimensions the i_n are unit vectors along the x_n orthogonal hyper axes. The i_n are unit hyper vectors. The a_n are hyper components of the vector a in a hyper space. In one, two, and three dimensions one may draw a visual representation. For dimensions higher than three I do not know how to draw a representation. That we can not will not be a serious drawback. At this point, for convenience, we should like to rewrite equations (2) and (3) of 1-3 in a slightly different form:

(9) $a \cdot r = m$

(10) $b \cdot r = n$

Here a and b are any vectors and m and n are scalars. By exactly the same scheme of solution as in the first pair we get for their solution:

(11) $r = -(n\breve{a} - m\breve{b})/ a \cdot \breve{b}$

1-6 Systems Solution

We start with as simple a system as possible, one for which the student knows the answer by several different methods.

Example 1

Solve the system.

$$2x + y = 5$$

$$3x - 2y = 4$$

These two equations may be factored into:

$$a \cdot r = 5$$

$$b \cdot r = 4$$

$a = 2i + j, \quad b = 3i - 2j, \quad r = xi + yj.$

$$\breve{a} = 2\breve{i} + \breve{j} = 2j - i = -i + 2j$$
$$\breve{b} = 3\breve{i} - 2\breve{j} = 3j + 2i = 2i + 3j$$
$$a \cdot \breve{b} = (2i + j)\cdot(2i + 3j) = 4 + 3 = 7$$

According to (11) of 1-5 r is given by:

$$rn = -(4(-i + 2j) - 5(2i + 3j))/7$$
$$r = (14i + 7j)/7 = 2i + 1j$$
$$r = \qquad = xi + yj$$

whence $x = 2$, $y = 1$.

In this simple problem we have gone to a lot of pains to illustrate each point in detail. However, we are not yet done with it. In solving this problem we were to take note of any suggestions that it might give for generalizing the technique to large systems.

To begin with we point out that one does not have to, nor as such should it be done, calculate the quantity $a \cdot \breve{b}$ in the denominator of the solution for it is already nearly calculated. We repeat the original, writ ing down omly the coefficients:

$$2 + 1 = 5$$
$$3 - 2 = 4$$

Reducing the right side to zero as in the first case we get the row vector:

$$(-7 + 14).$$

We now cancel from this row, vector the largest factor possible 7 or - 7, it makes no difference, and we get the row vector

$$(-1 + 2).$$

We replace each member in this row by its column cofactor getting:

$$(2 + 1).$$

On does not change the direction of a vector by dividing it by a scalar, such as $a \cdot \breve{b}$. In this case we choose :

$$x = 2 s$$

$$y = 1 s$$

Putting these values of x and y into our first equation we get

$$5 s = 5$$

$$s = 1$$

and our answers are: $(x, y) = (2, 1).$

In this case, we point out, we did not calculate $a \cdot \breve{b}$.

This is one interpretation of our equation (11) of § 1-5 . There are many others. One can write the answer mentally. We now ask the question: Is this column cofactor taking a generalization applicable to all systems, large or small. The answer is yes. We next illustrate with a system of three unknowns.

Example 2

Solve the system of equations:

$$\begin{vmatrix} 1 & +2 & -1 \\ 2 & 1 & -1 \\ -1 & -1 & 2 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$$

We subtract the second from twice the first and the third from three times the first, reducing the right side to zero, and we get the two row vectors.

$$\begin{matrix} 0 & 3 & -1 \\ 4 & 7 & -5 \end{matrix}$$

The column cofactors of these two rows from left to right are:

$$-8 \quad -4 \quad -12$$

We cancel the largest factor from these and we get:

$$(\, 2 \quad 1 \quad 3 \,)$$

Of course these are the answers sought, all at one time. If one wants to test them we could call them $x = 2s$, $y = s$, $z = 3s$ and put these values back into any one of the original equations and we get s equal to one. s may not always come out equal to one but in any case the answer will be correct.

We now apply the technique to a formulation for many variables. We may write our system symbollically as:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} & \cdots & a_{nn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_n \end{vmatrix} = \begin{vmatrix} a_{10} \\ a_{20} \\ a_{30} \\ a_{n0} \end{vmatrix}$$

This system may be factored into

$$a_1 \cdot r = a_{10}$$

$$a_2 \cdot r = a_{20}$$

$$\cdot \; \cdot \; \cdot \quad \cdot \quad \cdot$$

$$a_n \cdot r = a_{n0}$$

A symbolic solution for the last set of equations is

$$r = a_{10}\hat{a}_1 + a_{20}\hat{a}_2 + a_{30}\hat{a}_3 \cdots a_{n0}\hat{a}_n .$$

$$a_n = (\, a_{n1} , a_{n2} , a_{n3} \cdots a_{nn} \,).$$

$\hat{a}_n$ is the vector whose components from left to right are the column cofactors from left to right in the matrix of the original system with the n th row omitted. It is easy to show that each of the $\hat{a}_m$ is perpendicular to each of the other n - 1 vectors similarly formed.

If we stopped here we would defeat our purpose. Instead we rewrite our last system of equations in the form:

$$
\begin{aligned}
a_1 \cdot r &= a_{10} \\
A_2 \cdot r &= 0 \\
\cdot \;\cdot \;\cdot \;\; &\cdot \; 0 \\
A_n \cdot r &= 0
\end{aligned}
$$

The symbolic solution to this system is the single term

$$r = a_{10}\, \hat{a}_1$$

$$A_m = (a_{m0}\, a_1 - a_{10}\, a_m)$$

$\hat{a}_1$ in this case is the vector whose components are the column cofactors from left to right of the Cap A matrix in the last system. We also test for s values in a complex system if not obvious. We do a system with four variables as an example.

Example 3

Solve the system:

$$
\begin{bmatrix} 1 & -2 & 1 & 1 \\ 2 & 1 & -1 & -1 \\ -1 & 0 & 2 & 0 \\ -2 & 3 & -1 & -1 \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}
=
\begin{bmatrix} 2 \\ 3 \\ -4 \\ -3 \end{bmatrix}
$$

This may be written:

$$\begin{vmatrix} 1 & -2 & 1 & 1 \\ -1 & -8 & 5 & 5 \\ 1 & -4 & 4 & 2 \\ -1 & 0 & 1 & 1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} = \begin{vmatrix} 2 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

The column cofactors of the last three rows from left to right are:

$$-16, \quad -8, \quad 8, \quad -24$$

We take the largest common factor from these and get:

$$(\ 2, \quad 1, \quad -1, \quad 3\ )$$

which are the answers. They are too obvious to test for the s value.

The small equation $r = a_{1\cdot} \hat{a}_1$ is a BLUE RIBBON in systems solution, for which the illustrative examples testify.

There are many interpretations that may be given to a system of equations from the Mutation Viewpoint. We have given one of them here just to show the versatility of the New Science of Mutation Geometry. Geometry may not be the proper word here.

We have digressed a bit here from the main line of our Geometric exposition. We now resume it. We deem it time to make plausible the contents of the Mutation Proposition which is a proposition of dissolution, splintering products of the form:

$$a \cdot r \ b \cdot r$$

which are called omega products into a sum of alpha type products.

Before we launch into the simplicities of the only proposition in Mutation Geometry we present a list of exercises to occupy the students in their exciting journey of discovery. They are to be done strictly from the Mutation Standpoint.

Exercises

Solve the following systems of equations, using the Mutation scheme alone.

1 $$\begin{aligned} 3x + 2y &= 5 \\ 2x + y &= 3 \end{aligned}$$

2 $$\begin{aligned} 2x - 3y &= 1 \\ 3x + 2y &= 8 \end{aligned}$$

3 $$\begin{aligned} 5x + y &= 3 \\ -2x + 4y &= 1 \end{aligned}$$

4 $$\begin{aligned} 4x - 3y &= 6 \\ 2x - 5y &= -4 \end{aligned}$$

5 $$\begin{aligned} x + y &= 5 \\ x - y &= 1 \end{aligned}$$

6 $$\begin{aligned} 6x + y &= 2 \\ 3x - 4y &= -8 \end{aligned}$$

7 $$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ -1 & -1 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 5 \\ 7 \\ -2 \end{vmatrix}$$

8 $$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} -1 \\ 6 \\ 3 \end{vmatrix}$$

9 $$\begin{vmatrix} 1 & -2 & 2 \\ 2 & 1 & 3 \\ -1 & -1 & -1 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} -3 \\ 5 \\ -4 \end{vmatrix}$$

10 $$\begin{vmatrix} 8 & 3 & 4 \\ 4 & 6 & -4 \\ -4 & -3 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 5 \\ 1 \\ -1 \end{vmatrix}$$

11 $$\begin{vmatrix} 6 & 2 & 5 \\ 3 & 4 & -10 \\ -9 & 6 & 5 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 8 \\ -2 \\ 0 \end{vmatrix}$$

12 $$\begin{vmatrix} 1 & 2 & -2 \\ 2 & 0 & 1 \\ 0 & 3 & 2 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 3 \\ 1 \\ -2 \end{vmatrix}$$

1-7 Omega Proposition

The first demonstration of the Mutation Proposition will be for the plane where most of our interest lies. The second demonstration, for completeness, will be for a hyper-space of any number of dimensions. See the sketch below for the plane.

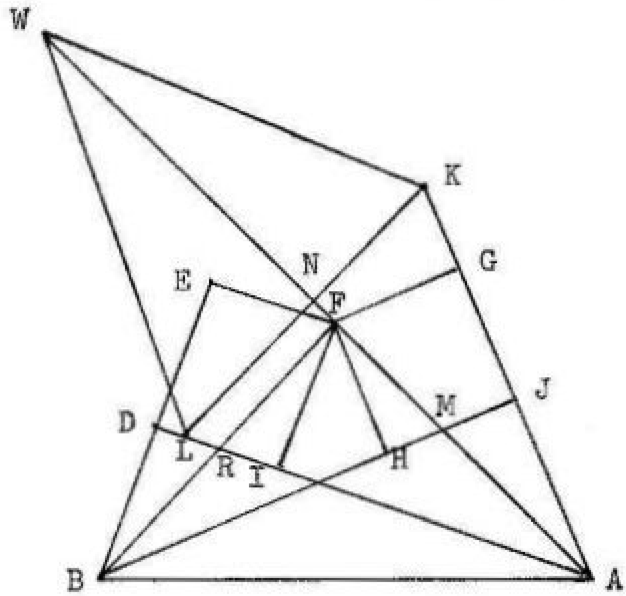

Fig. 1-8

We first renew our statement of the MUTATION PROPOSITION, a proposition of dissolution, splintering products of the form: $a \cdot r\, b \cdot r$, called omega products , into a sum of single products of the form: $a \cdot s$, $b \cdot p$, ... called alpha products.

Given any two vectors a and b and any unit vector r we shall show that:

(1) $$a \cdot r\, b \cdot r = (a \cdot b + a \cdot \gamma)/2$$

(2) $$\gamma \wedge b\, r , \quad \gamma = b_o$$

Equation (2) states that gamma is the symmetric of b with respect to r and in magnitude is equal to b. This is an equational statement of the PROPOSITION OF MUTATION.

The Mutation Diagram $\gamma \wedge b\, r$ is a consequence of the Mutation Proposition.

Proof: See Fig. 1 - 8 for drawing and lettering. Let AB represent a and AK b . Let the unit direction r lie along AW. Draw a line AL equal to AK and making angle KAW equal to angle WAL. Complete the rhombus on AK and AL as KALW. Draw BD, BF, and BJ perpendicular to AL, AW, and AK respectively. Draw FE, FI, FH, and FG perpendicular to BD, AL, BJ, and AK respectively. N is the common point to lines AW and KL. M is the common point to lines AW and BJ. It will now be shown that BF is the bisector of angle EBH. In right triangles BFM and AMJ there is a common angle at M so angle FBM and MAJ are equal. In right triangles BDR and AFR there is a common angle at R so angle RBD and RAF are equal. Now angle RAF and MAJ are equal by construction. Therefore angle RBD and FBM are equal. The following relations may be written:

(3) $$JG = HF = FE = DI$$

(4) $$FI = FG$$

(5) $$AG = AI$$

(6) $$\begin{aligned} AG &= AJ + JG = AJ + DI \\ &= AJ + AD - AI \\ &= AJ + AD - AG \end{aligned}$$

(7) $$2\,AG = AJ + AD$$

Multiply both sides of equation (7) by AK and we obtain:

(8) $$2\,AK\,AG = AK\,AJ + AK\,AD$$

From the similar right triangles AFG and ANK one has:

(9) $$AK / AF = AN / AG$$

(10) $$AK\,AG = AF\,AN$$

Put (10) into (8) and we get the following result:

(11) $2\ AF\ AN = AK\ AJ + AK\ AD$

(12) $AF = a \cdot r, \quad AN = b \cdot r$

(13) $AK\ AJ = a \cdot b$

(14) $AK\ AD = AL\ AD = a \cdot \gamma$

Here AL, equal to AK, equal to b in magnitude, is represented by γ.

Put (12), (13), and (14) into (11) and we get:

(15) $a \cdot r\ b \cdot r = (a \cdot b + a \cdot \gamma)/2$ QED.

Equation fifteen represents the MUTATION PROPOSITION.

This demonstration is for two dimensions. We now show it for a hyper-space of any number of dimensions. For two or three dimensions one can draw a representation for it but for a hyper-space this is not easy (I do not know how) to draw. For most geometry of interest, plane geometry, our proposition for the planm will be adequate.

For a hyper-space, when γ is the symmetric of b with respect to r, we may write:

(16) $$a \cdot r b \cdot r = (a \cdot (b + \gamma)/(b + \gamma)_0) (b \cdot (b + \gamma)/(b + \gamma)_0$$

$$= a \cdot (b + \gamma)\ b \cdot (b + \gamma)/(b + \gamma)^2$$

$$= \frac{(a \cdot b + a \cdot \gamma) (b^2 + b \cdot \gamma)}{b^2 + 2 b \cdot \gamma + b^2}$$

$$= \frac{(a \cdot b + a \cdot \gamma) (b^2 + b \cdot \gamma)}{2 (b^2 + b \cdot \gamma)}$$

$$= (a \cdot b + a \cdot \gamma)/2 \qquad \text{QED}$$

If we had chosen γ to be symmetric of a with respect to r instead of with respect to b our equation would be written

(17) $a \cdot r\ b \cdot r = (a \cdot b + b \cdot \gamma)/2$

In the first case the mutation diagram will read:

$\gamma \wedge$ br

In the second case it will read:

$\gamma \wedge$ ar

It is at the discretion of the operator into which entities he sends the splintered gammas from the dissolution of a number of omega products. At times it may be convenient to send or disperse the gammas in various groups into various primordial prototypes or mental corral. If I were interested in only a single gamma I would send them all into a single corral. They all become one there and I do not have to be confused in identifying who is who. If I were interested in certain relations among the gammas I would send them accordingly. For instance, we might have seven omegas to splinter and I wanted to know a relation between three particular gammas. In this case I would hold the three gammas in their own corrals and herd the remaining four gammas into these corrals. It is obvious that it can be done in several ways. Let us suppose that out of the seven gammas mentioned above we are interested in gammas 1, 4, and 6. As one choice we might send 3 and 5 into 4 and 2 and 7 into 6. Into whatever primordial prototype we may send the various gammas the MUTATION DIAGRAM will pinpoint the corresponding co-migrates. In the end we shall have a relation between gammas 1, 4, and 6.

Conventional mathematics, as far as I know, has no such power.