CHAPTER TWO

2 - 1 The Straight Line

In chapter 1 we wrote the equation of a straight line as:

$$(1) \qquad a' \cdot r = a_0$$

Let us now find the equation of a line thru two given points as a (a_1 , a_2) and b (b_1 , b_2). See the sketch below. Let **r** be a vector from the origin O to any point on the line thru the end points of a and b.

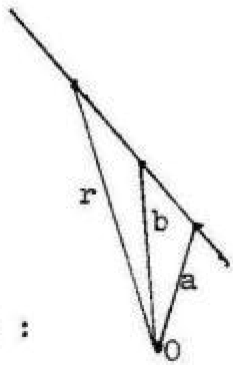

Fig. 2-1

$$(2) \qquad \breve{(a - b)} = \breve{a} - \breve{b}$$

is a line perpendicular to this line and

$$(3) \qquad (r - a)$$

is along the line. From (2) and (3) we get :

$$(4) \qquad (\breve{a} - \breve{b}) \cdot (r - a) = 0$$

or

$$(5) \qquad (\breve{a} - \breve{b}) \cdot r = -a \cdot \breve{b} = \breve{a} \cdot b$$

Examlpe 1

Find the equation of the line thru a (3 , 2) and b (1, 5)

$$(a - b) = (a_1 - b_1)\,i + (a_2 - b_2)\,j$$

$$(a - b)^{\smile} = (a_1 - b_1)\,\breve{i} + (a_2 - b_2)\,\breve{j}$$

$$= (a_1 - b_1)\,j - (a_2 - b_2)\,i$$

$$= (b_2 - a_2)\,i + (a_1 - b_1)\,j$$

$$\breve{a} \cdot b = a_1 b_2 - a_2 b_1 = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Equation (5) now becomes:

$$(6) \qquad (b_2 - a_2)\,x + (a_1 - b_1)\,y = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

We now write down our coordinates in a rectangular array:

$$\begin{matrix} 3 & 2 \\ 1 & 5 \end{matrix}$$

We get the coefficient of x by subtracting the 2 from the 5 and the coefficient of y by subtracting the 1 from the 3 and the absolute term is the determinant of the array thus we can write the equation mentally:

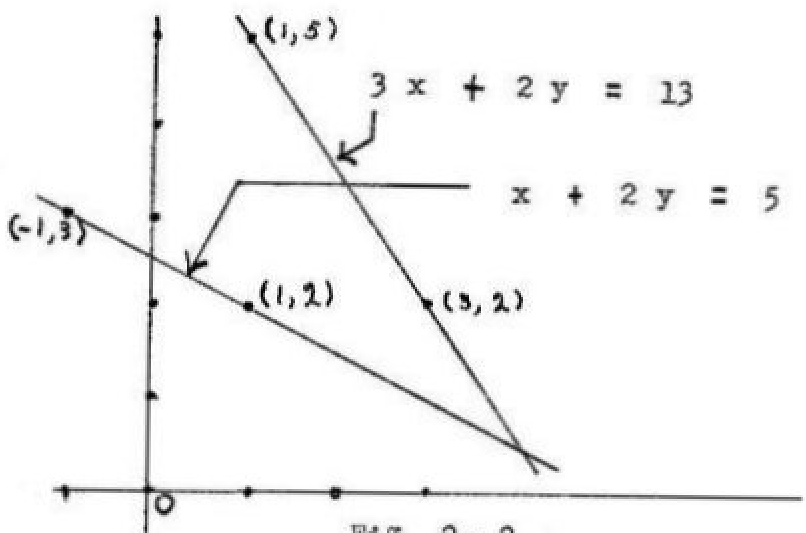

Fig. 2 - 2

Example 2

Find the equation of a line thru the points (1, 2) and (- 1, 3) See sketch above.

We simply write the points down one above the other and write down the answer:

1	2
-1	3

$$x + 2y = 5$$

One conventional geometry way to solve this problem is to compare the slopes of different segments of the same line which are the same. See the shetch below.

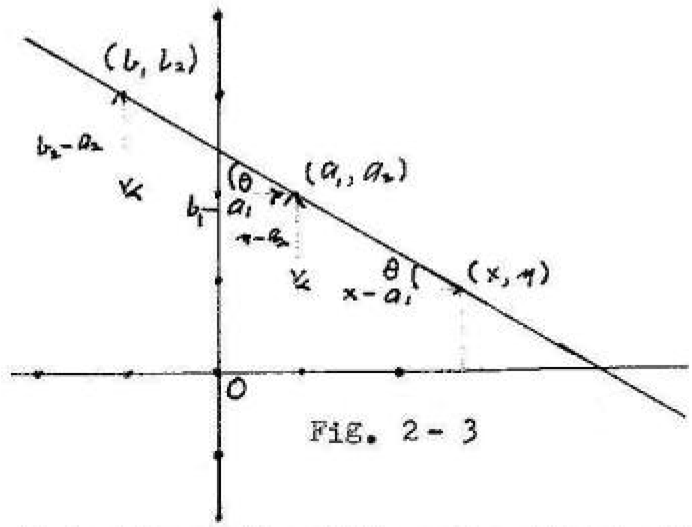

Fig. 2- 3

Let θ be the angle which each segment of the line makes with the x axis then we may write the following equations:

$$(y - a_2)/(x - a_1) = (b_2 - a_2)/(b_1 - a_1) = \tan\theta$$

Putting in the values of the coordinates we get

$$(y - 2)/(x - 1) = (3 - 2)/(-1 - 1)$$

Simplifying this we get:

$$x + 2y = 5.$$

Another conventional way of doing the problem is to assume an equation of a straight line as :

$$A x + B y = C$$

Put each set of coordinates into this equation which they must satisfy since the points are on the line and we get:

$$A a_1 + B b_2 = C$$

$$A b_1 + B b_2 = C$$

Solving these two equations for A and B we get:

$$A = C (b_2 - a_2)/ (a_1 b_2 - a_2 b_1)$$

$$B = C (a_1 - b_1)/ (a_1 b_2 - a_2 b_1)$$

Putting in the numerical values of the coordinates we get

$$A = C/5$$

$$B = 2C/5$$

Putting these values of A and B back into the assumed equation and we get:

$$(C/5) x + (2 C/5) y = C$$

Cancelling the C and simplifying we get

$$x + 2 y = 5 .$$

as before. Compare these solutions with the smoothe styling of the Mutation Solution.

Equation of a Line with Given Intercepts. 2 - 2

If the line cuts off intercepts of a and b on the x and y axes respectively then two points on the line are (a, 0) and (0, b). To get the equation of a line thru these two points do as in the previous example and write

$$\begin{matrix} a & 0 \\ 0 & b \end{matrix}$$

and write down the equation

$$b\,x + a\,y = a\,b$$

or in the more usual form:

(1) $$x/a + y/b = 1$$

See the sketch below.

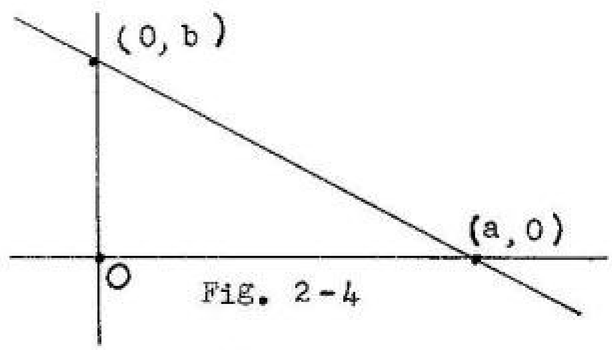

Fig. 2-4

Exercises

Write the equations of the lines thru the following points:

1	(2, 0), (0, 5)	2	(6, 0), (0, 3)
3	(1, 4), (3, 3)	4	(-1, 7), (4, 6)
5	(-2, 5), (-7, 4)	6	(3, 2) , (6, 2)

Any straight line equation may be reduced to the intercept form. For example we take the general equation

$$A x + B y = C$$

Dividing this by C we get in the form

$$x/(C/A) + y/(C/B) = 1$$

which is in the intercept form, the intercepts being

C/A and C/B

As an illustrative numerical we put the equation

$$4 x + 3 y = 2$$

into intercept form. It is

$$x/ (1/2) + y/ (2/3) = 1$$

Exercises

Put the following equations in intercept form:

1b	$4 x + 5 y = 4$	2b	$3 x + 2 y = 9$
3b	$2 x + 1 y = 7$	4b	$6 x + 3 y = 2$
5b	$-7 x + 4 y = 8$	6b	$-1 x + 8 y = 4$
7b	$x/2 + y/3 = 5$	8b	$-x/3 + 2y = 3$

2-3 Normal Form of a Line

The first form of the equation of a line which we wrote in this book is called the normal form. It is

(1) $$a' \cdot r = a_0$$

We point out that a_0 is the perpendicular distance from the origin and a' was the unit vector along that perpendicular.

We now show how to put any equation of a straight line in the normal form. Suppose we have any equation

$$a_1 x + a_2 y = S$$

This may be written

$$a \cdot r = S$$

where

$$a = a_1 i + a_2 j$$
$$r = x i + y j$$
$$a = a_0 a'$$
$$a' = a/a_0$$
$$a_0 = \sqrt{a_1^2 + a_2^2}$$

We now divide each side of our above equation by a_0 and get

$$(a/a_0) \cdot r = S/a_0$$

which is the same thing as

$$a' \cdot r = S/a_0$$

which is the same thing as

(2) $$((a_1 i + a_2 j)/\sqrt{a_1^2 + a_2^2}) \cdot r = S/\sqrt{a_1^2 + a_2^2}$$

which is the same as

(3) $$(a_1 x + a_2 y)/\sqrt{a_1^2 + a_2^2} = S/\sqrt{a_1^2 + a_2^2} .$$

This last equation is the usual form of the normal equation. We have gone to a lot of pains to show just why one simply divides both sides of the equation by the magnitude of the coefficient vector which is the square root of the sum of the squares of the coefficients of x and y.

We point out that

(4) $$a' = (a_1 i + a_2 j)/\sqrt{a_1^2 + a_2^2}$$

is a unit vector for if we square it we get unity. It should also be noted that when the equation is in the normal form and written so that the right hand side is postive then the right hand member represents the perpendicular distance fron the origin to the line and the a in this case points from the origin toward this line.

It is important then that we write our equations in the normal form so that the right hand member is positive or the equivalent of it. It will avoid a lot of confusion. Some of the older and even recent text books on analytic geometry are slightly hazy and confused on this topic.

Example 1

See the sketch below. Put the following equation in normal form and find the directional distance of this line from the origin.

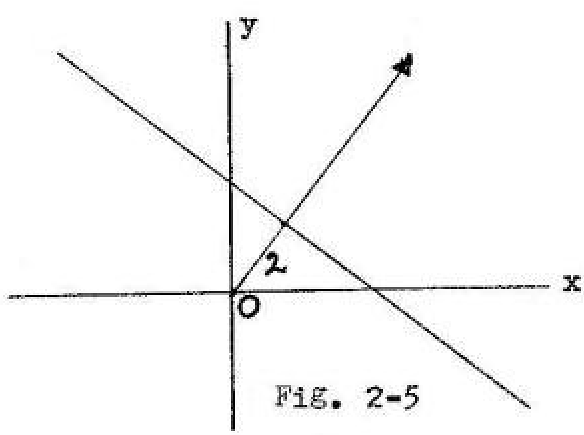

Fig. 2-5

1 $$3x + 4y = 10$$

Divide each side of this equation by 5 and we get it in the normal form:

2 $$(3x + 4y)/5 = 2$$

The distance from the origin to this line is 2 units and it lies along the vector

3 $$3\,i + 4\,j$$

Example 2

See the sketch below. Put the following equation in normal form and find its directional distance from the origin.

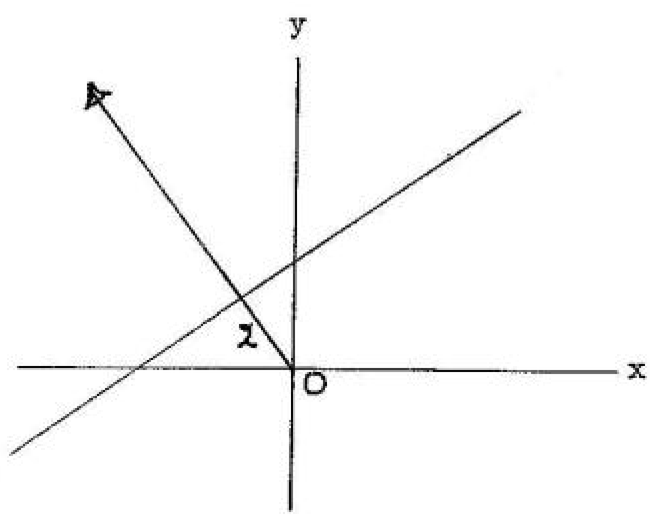

Fig. 2 - 6

1 $$3\,x - 4\,y = -10$$

The normal form of this equation is

2 $$(-3\,x + 4\,y)/5 = 2$$

Notice that the equation has been changed so that the right member is positive. The perpendicular distance from the origin to this line is 2 units and it lies along the vector

3 $$-3i + 4j$$

It is properly pictured in Fig. 2 — 6

Example 3

See the sketch below. Put the following equation in normal form and find it's directional distance from the origin.

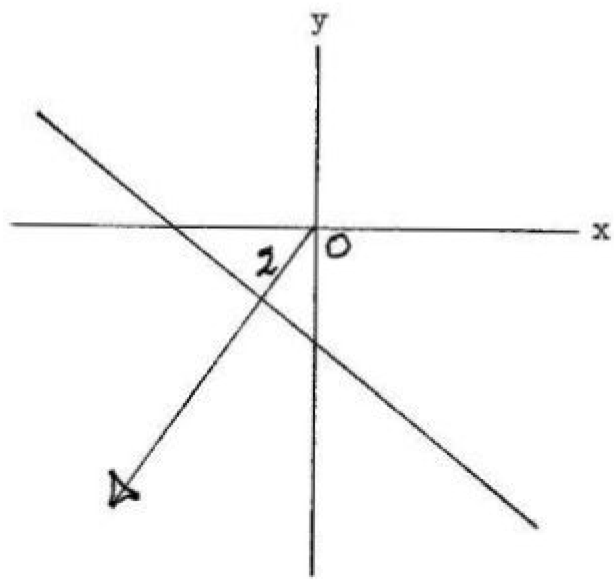

Fig. 2 - 7

1 $$3x + 4y + 10 = 0$$

The normal form of this equation is

2 $$(-3x - 4y)/5 = 2$$

Notice that it has been changed so that the right member is postive. The distance from the origin is 2 units and it lies along the vector

3 $$-3i - 4j$$

Example 4

See the sketch below. Put the following equation in normal form and find its directional distance from the origin.

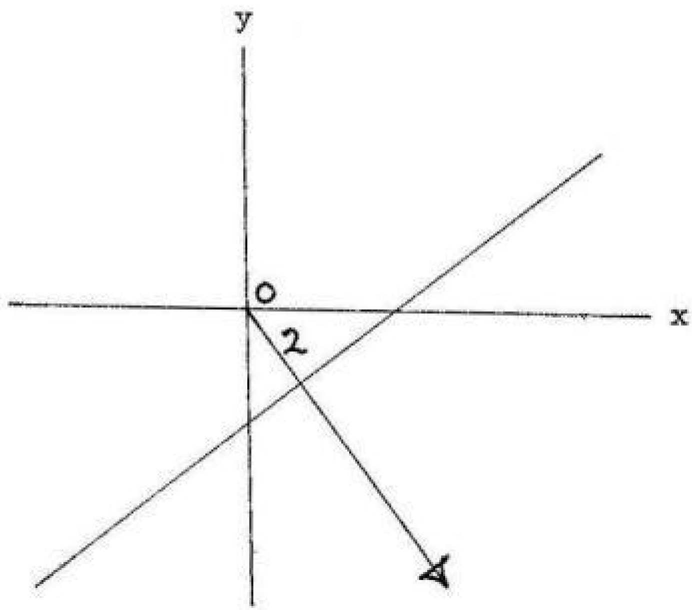

Fig. 2 - 8

1 $$3x - 4y = 10$$

The normal form of this equation is

2 $$(3x - 4y)/5 = 2$$

Its distance from the origin is 2 units and it lies along the direction of the vector

3 $$3i - 4j$$

This vector points into the fourth quadrant . It is correctly pictured above.

Henceforth we shall call the perpendicular distance from the origin to a line p . Then our normal equation may be written:

$$(5) \qquad a' \cdot r = p$$

In the last four illustrations we have taken an easy and simple form in order to make the principle and procedure clear to the student. For a final illustration on this topic we use a casual equation.

Example 5

See the sketch below. Put the following equation in the normal form and find its directional distance from the origin.

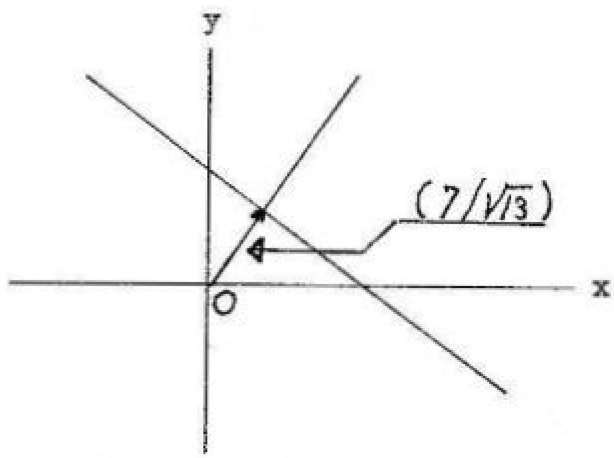

Fig. 2 - 9

1 $$2x + 3y = 7$$

The normal form of this equation is

2 $$(2x + 3y)/\sqrt{13} = 7/\sqrt{13}$$

2 - 4 Distance From a Given Point to a Given Line

See the sketch below.

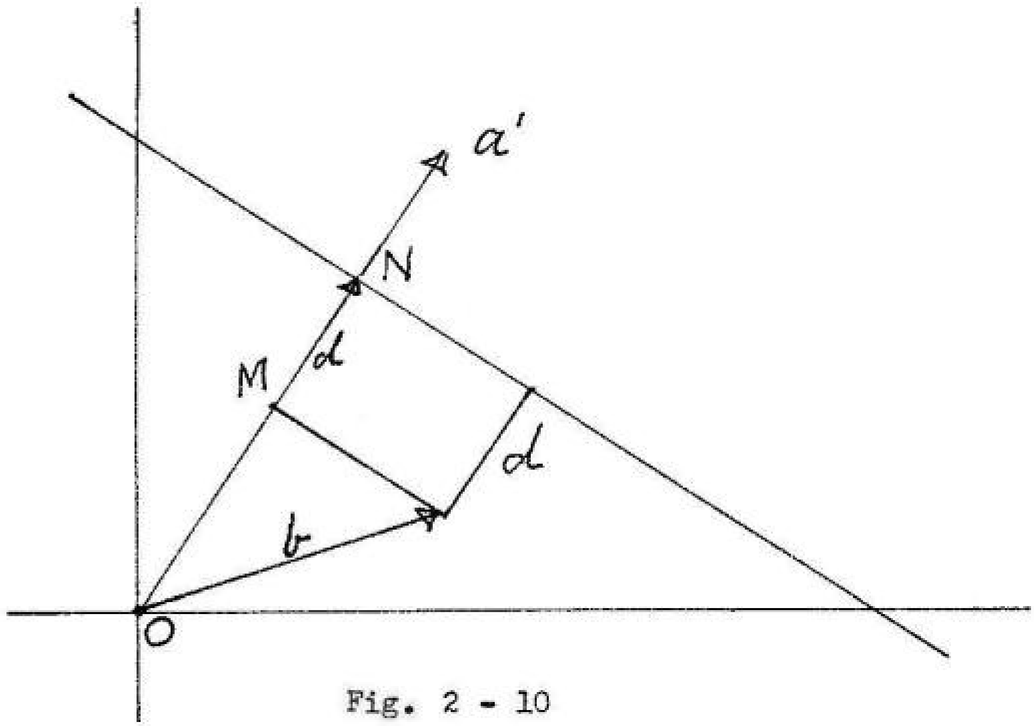

Fig. 2 - 10

Let the equation of the given line be

$$(1) \qquad a' \cdot r = p$$

and the given point be represented by

$$(2) \qquad b = b_1 i + b_2 j = b(b_1, b_2)$$

Draw a perpendicular from the end of b to the given line. Call this distance d . Let N designate the foot of the perpendicular p with the given line. Let M designate the foot of the perpendicular of a line from the end of b to the line p. Then

$$(3) \qquad d = MN = ON - OM = p - b \cdot a'$$

$$a' = (a_1 i + a_2 j)/\sqrt{a_1^2 + a_2^2}$$

$$b = b_1 i + b_2 j$$

$$a' \cdot b = (a_1 b_1 + a_2 b_2)/\sqrt{a_1^2 + a_2^2}$$

$$(4) \quad d = p - (a_1 b_1 + a_2 b_2)/\sqrt{a_1^2 + a_2^2}$$

p, a known quantity, is always the right member of the normalized given equation . a_1 and a_2 are known components. the b_1 and b_2 are given quantities so everything in (4) is known. If d comes out negative it just means that the given point was on the far side of the line from the origin. Incidentally, one may test by this means whether a point is on the near or far side of a line from the origin.

One sees from (4) that the b_1 and b_2 simply replace the x and the y in the given equation.

Example 1

See the sketch below.

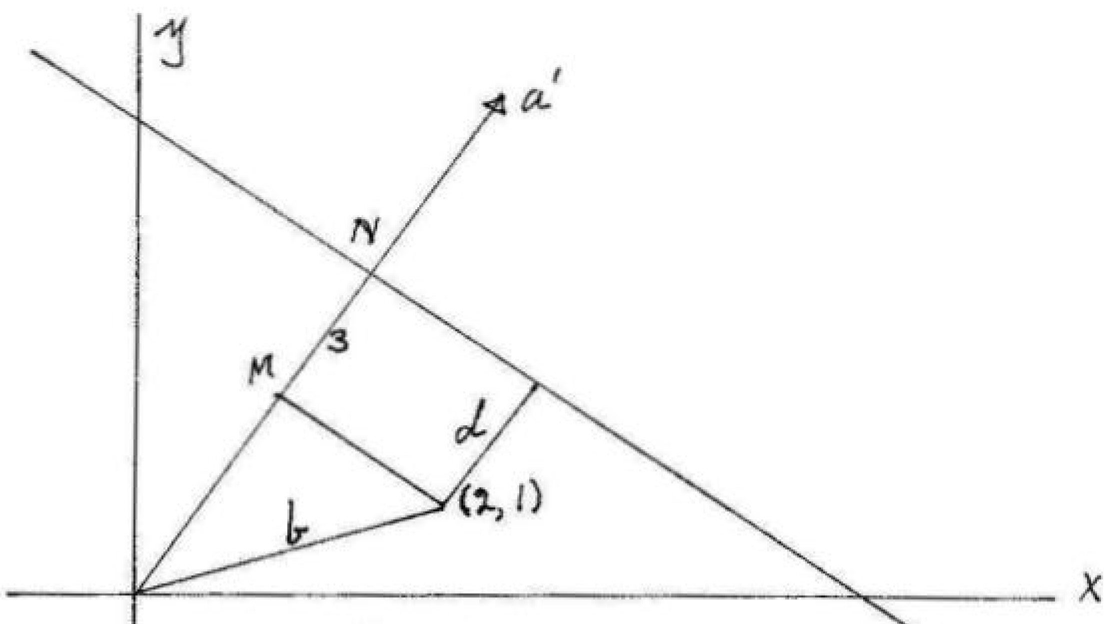

Fig. 2 - 11

Find the distance from the point b (2, 1) to the equation.

1 $$3x + 4y = 25$$

The normal form is

2 $$(3x + 4y)/5 = 5$$

Here p is 5. and

3 $$a' \cdot b = (3(2) + 4(1))/5 = 10/5 = 2$$

4 $$d = p - a' \cdot b = 5 - 2 = 3$$

We have gone to a lot of detail just to make it plain to the student just how every piece fits into the picture.

Example 2

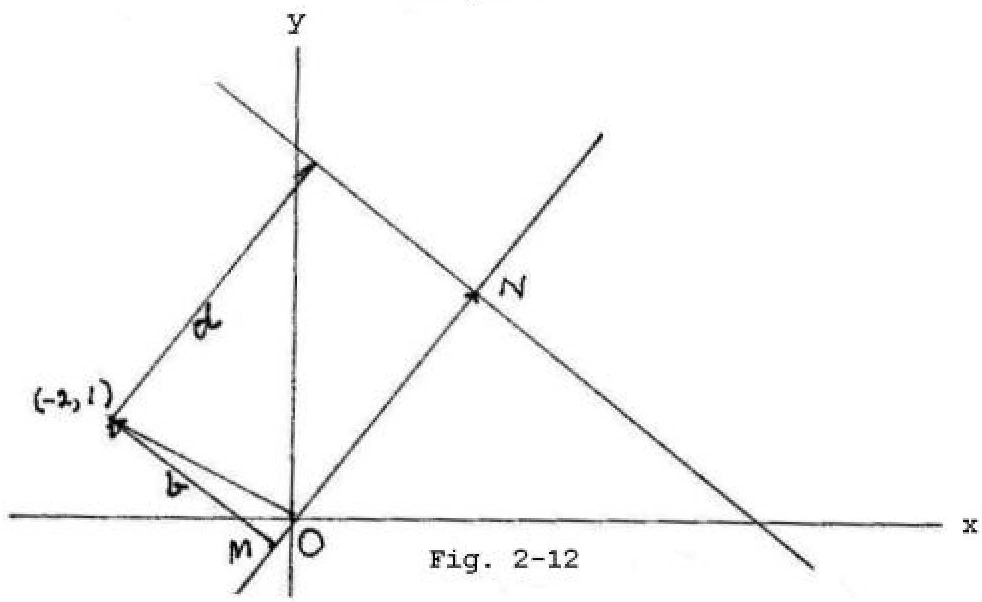

Fig. 2-12

Find the distance from the point b (-2, 1) to the line given by

1 $$3x + 4y = 15$$

2 $$p = 15/5 = 3$$

3 $$a' \cdot b = (3(-2) + 4(1)/5 = -2/5 = -0.4$$

4 $$d = p - a' \cdot b = 3 - (-0.4) = 3.40$$

Example 3

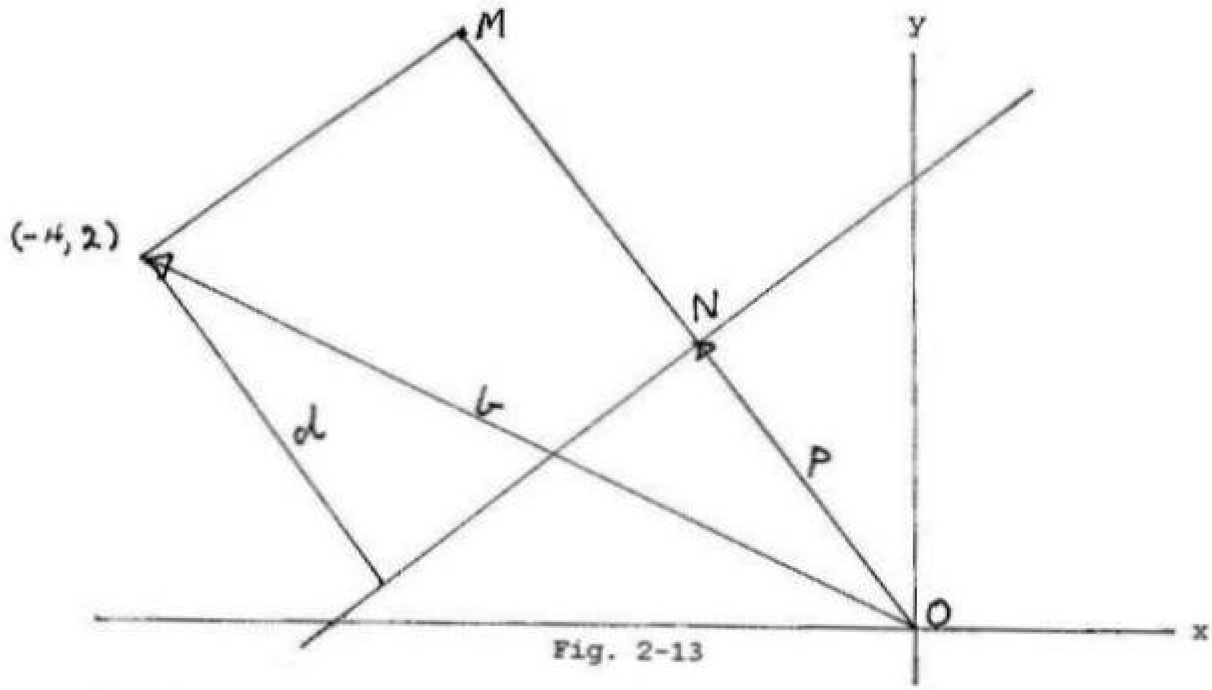

Fig. 2-13

See Fig. 2-13. Find the distance from the point b (-4, 2) to the line whose equation is

(1) $$-3x + 4y = 10$$

(2) $$p = 10/5 = 2$$

(3) $$a' \cdot b = (-3(-4) + 4(2))/5 = 20/5 = 4$$

(4) $$d = p - a' \cdot b = 2 - 4 = -2$$

Here, the point is on the far side of the line from the origin.

Example 4

See Fig. 2-14 . Find the distance from the point b(-4, -2) to the line whose equation is

(1) $$-3x - 4y = 15.$$

2 $p = 15/5 = 3$

3 $a' \cdot b = (-3(-4) - 4(-2))/5 = 20/5 = 4$

4 $d = p - a \cdot b = 3 - 4 = -1$

Here the point is on the far side of the line from the origin.

Solve the same problem when the point is at b (4, 2). Here

1 $p = 15/5 = 3$

2 $a' \cdot b = \qquad = -4$

3 $d = 3 - (- 4) = 7$

Fig. 2-14 serves for both cases.

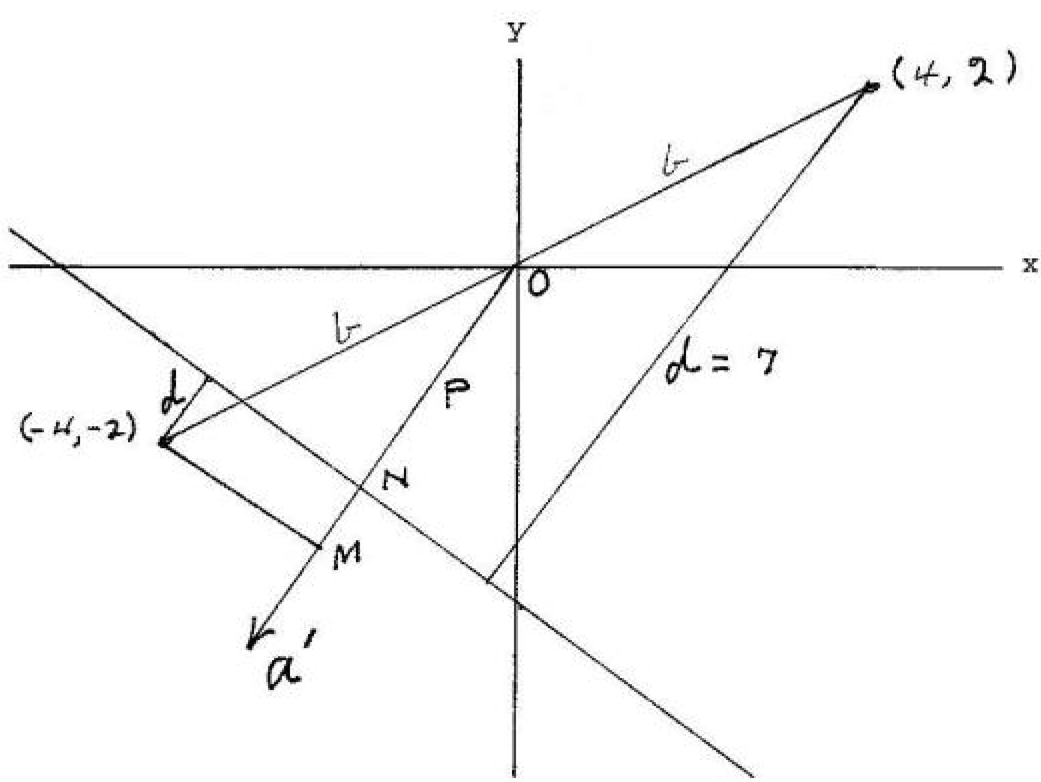

Fig. 2-14

Example 5

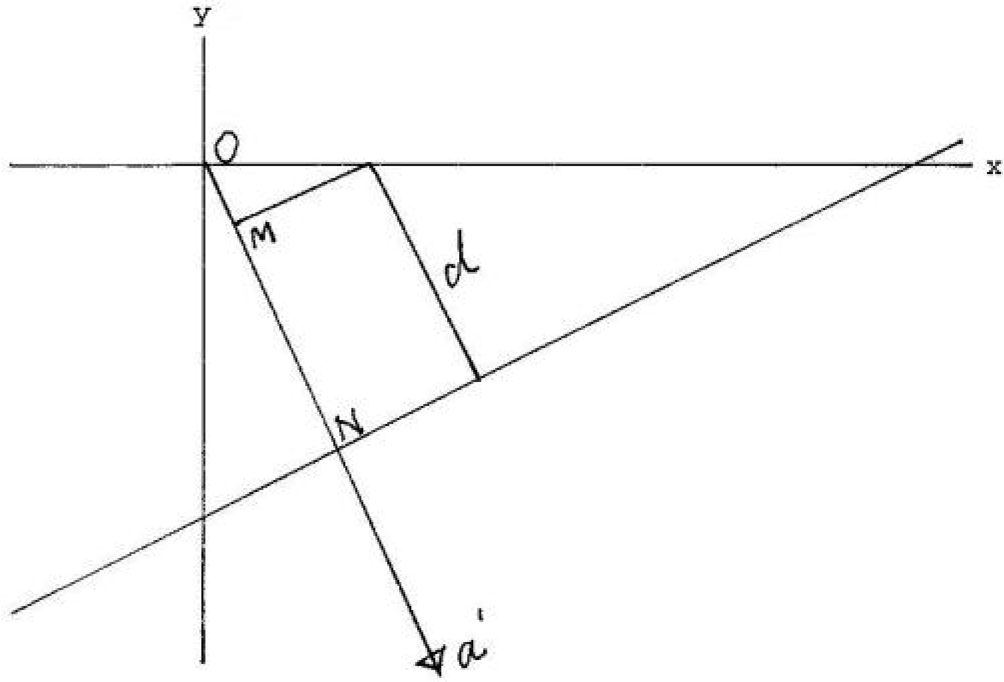

Fig. 2-15

See Fig. 2-15 For a final example on this topic it is required to find the distance from the point b (1, 0) to the line whose equation is

1 $$x - 2y = 4$$

2 $$p = 4/\sqrt{5}$$

3 $$a' \cdot b = 1/\sqrt{5}$$

4 $$d = 4/\sqrt{5} - 1/\sqrt{5} = 3/\sqrt{5}$$

Exercises

Find the directional distances of the following lines fron the origin.

1 $$6x + 8y = 10$$

2 $$5x + 12y = 39$$

3 $$8x + 15y = 68$$

4	2x + y = 8
5	3 x - 2y = 5
6	7 x - 8 y = 2
7	8 x + 6 y = 30
8	-12 x - 5 y = 65
9	3 x - 0 y = 6
10	15 x + 8 y = 51

In the following sets of lines and points find the distance from the point to its associated line:

1b	(2, 1)	6 x + 8 y = 20
2b	(1, 2)	4 x - 3 y = 35
3b	(0, 4)	2 x - 5 y = 8
4b	((4,-3)	5 x + 1y = 3
5b	(-2, 5)	8 x + 2 y = 7
6b	(2, 0)	3 x - 9 y = 6
7b	(-1,-1)	x + 2 y = 11
8b	(3, 2)	6 x - 5 y = 12
9b	(4, 7)	3 x + 6 y = 10
10b	(5, 1)	2 x + 4 y = 5

In the following set of lines and points state whether the point is on the near or far side of the line from the origin.

1 c	(2, 1)	6 x + 8 y = 40
2 c	(1, 1)	5 x + 12 y = 39
3 c	(4, 3)	4 x + 5 y = 10
4 c	(2, 1)	2 x + 3 y = 7

2 - 5 Angle Between two Lines

See Fig. 2-16. Let the equations of the two lines be given by

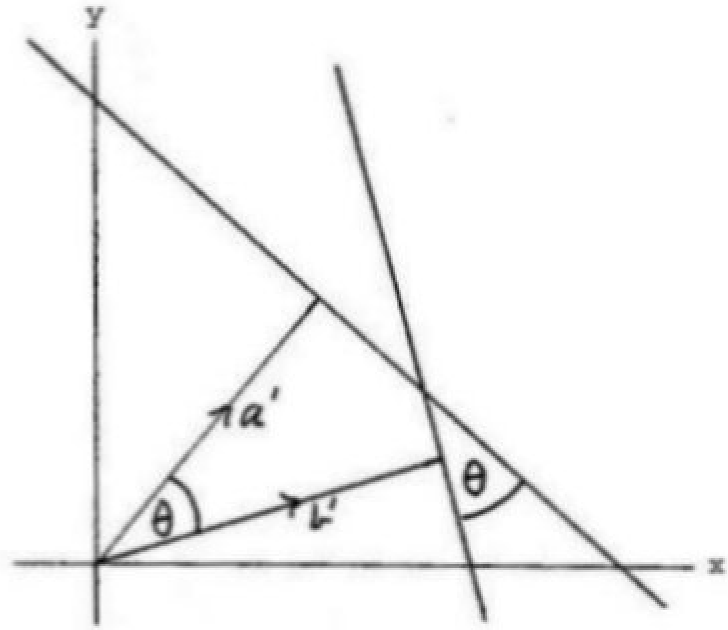

Fig. 2-16

1 $$a' \cdot r = p$$

2 $$b' \cdot r = s$$

Here a' and b' are the unit normals to the lines and the angle between the lines is the same as the the angle between their normals. So,

3 $$a' \cdot b' = \cos\theta$$

4 $$a' = (a_1 i + a_2 j)/\sqrt{a_1^2 + a_2^2}$$

5 $$b' = (b_1 i + b_2 j)/\sqrt{b_1^2 + b_2^2}$$

6 $$\cos\theta = \frac{(a_1 i + a_2 j)\cdot(b_1 i + b_2 j)}{\sqrt{(a_1^2 + a_2^2)(b_1^2 + b_2^2)}}$$

$$= \frac{a_1 b_1 + a_2 b_2}{\sqrt{(a_1^2 + a_2^2)(b_1^2 + b_2^2)}}$$

2 - 6 Equation of the Bisector of the Angle between two Given lines

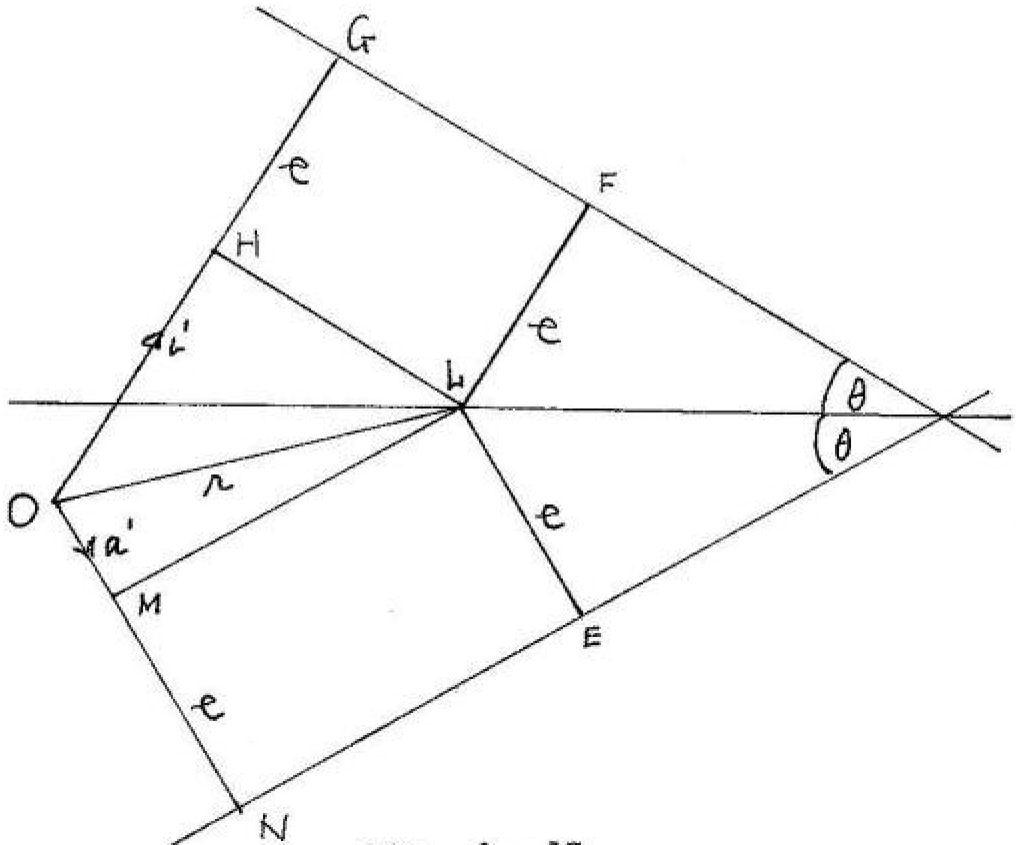

Fig. 2 - 17

See Fig. 2-17. In the first case let the origin O be inside the angle to be bisected. Let the equations of the given lines be

$$a' \cdot r = p$$

$$b' \cdot r = s$$

Let the perpendiculars, p and s, from O meet the given lines in N and G respectively. Let r be the vector to a point on the bisector of the given angle. From the end of r draw perpendiculars to p and s meeting them in M and H respectively.

From L the end of r draw perpendiculars to the given lines meeting them in E and F respectively. Let e be the magnitude of the lines LE and LF since they are equal. From the figure we have

$$e = LE = MN = p - OM = p - a' \cdot r$$

$$e = LF = HG = s - OH = s - b' \cdot r$$

$$p - a' \cdot r = s - b' \cdot r$$

$$(1) \quad (a' - b') \cdot r = p - s$$

Equation (1) is the equation of the bisector when the origin is inside the angle to be bisected. For a sketck when the origin is outside the angle to be bisected see Fig. 2-18.

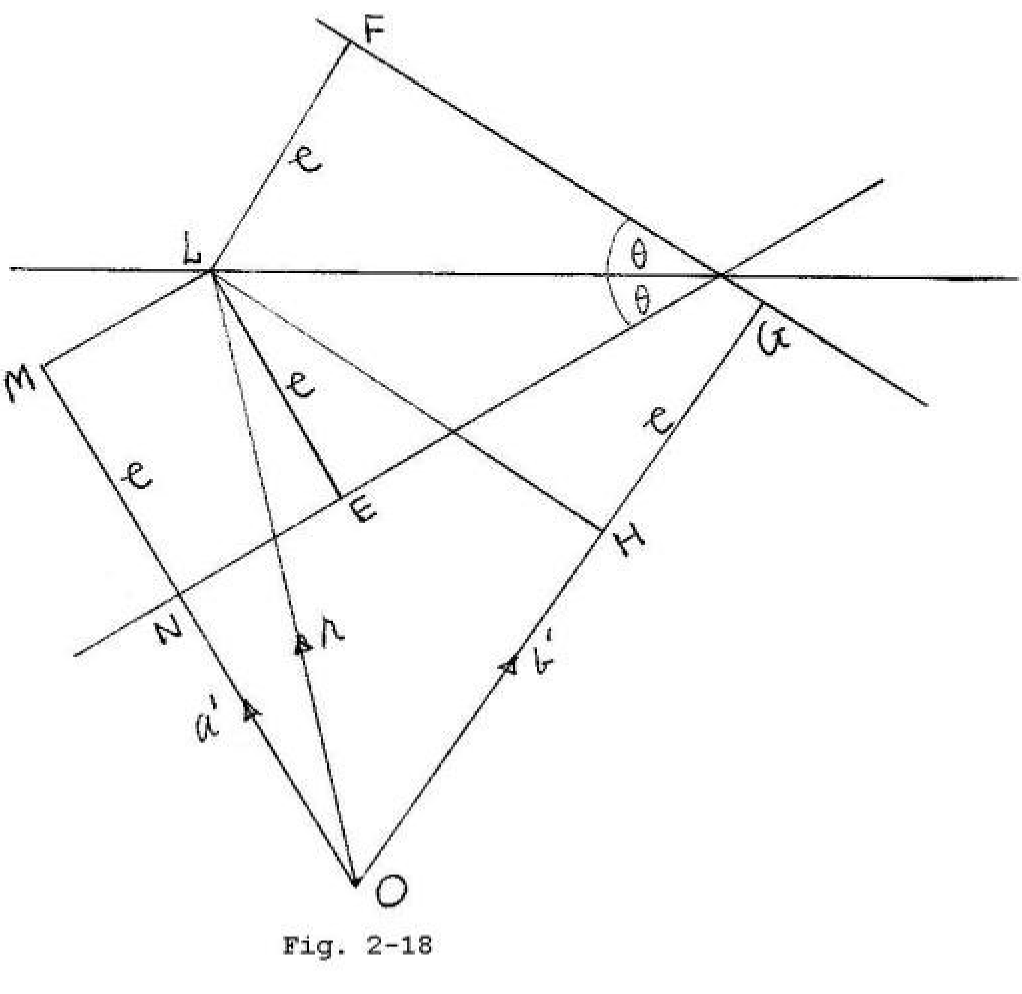

Fig. 2-18

In this case we have the following relations:

$$e = LE = MN = OM - ON = a' \cdot r - p$$

$$e = LF = HG = OG - OH = s - b' \cdot r$$

$$a' \cdot r - p = s - b' \cdot r$$

$$(2) \quad (a' + b') \cdot r = p + s$$

Notice the difference in the signs in equation (1) and (2). We are now in a position to specify beforehand what angle we want to bisect and to write its equation .

Example 1

Find the equation of the bisector if the angle in which the origin does not lie formed by the two lines

$$-3x + 4y = 2$$

$$13x + 5y = 3.$$

$$a' = (-3i + 4j)/5$$

$$b' = (12i + 5j)/13$$

$$p = 2/5$$

$$s = 3/13$$

$$a' + b' = (21i + 77j)/65$$

$$p + s = 41/65.$$

$$21x + 77y = 41$$

is the required equation. It is the bisector of the angle in which the origin does not lie.

Example 2

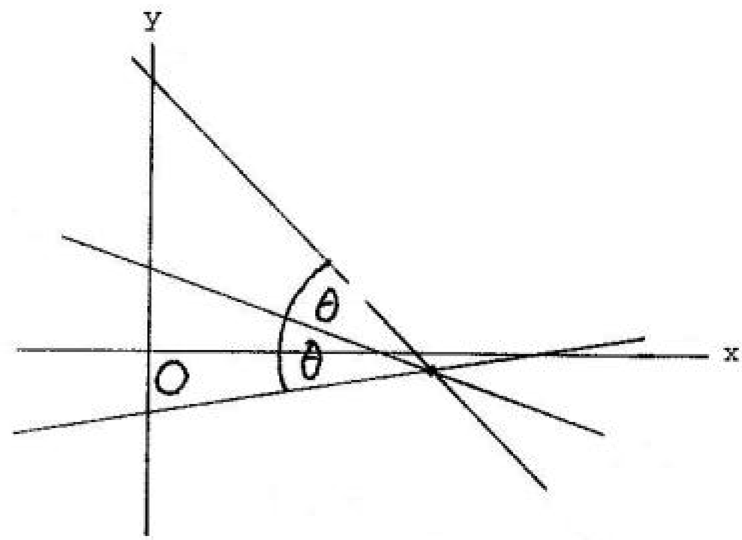

Fig. 2-19

See Fig. 2-19. Find the equation of the bisector of the angle, in which the origin lies, formed by the two lines whose equations are:

$$x + y = 3$$
$$x - 7y = 5.$$

$$a' = (i + j)/\sqrt{2}$$
$$b' = (i - 7j)/5\sqrt{2}$$
$$p = 3/\sqrt{2}$$
$$s = 5/5\sqrt{2} = 1/\sqrt{2}$$
$$a' - b' = (4i + 12j)/5\sqrt{2}$$
$$p - s = 2/\sqrt{2}$$

$$2x + 6y = 5$$

is the required equation. It is the equation of the bisector of the angle in which the origin lies

Exercises

Find the equation of the bisector of the angle in which the origin lies for the following pairs of lines:

1. $3x - 4y = 7$
   $15x + 8y = 6$
2. $12x - 5y = 3$
   $8x + 6y = 2$
3. $7x - 1y = 5$
   $1x + 1y = 4$
4. $8x - 15y = 9$
   $5x + 12y = 1$
5. $2x - 2y = 5$
   $1x + 7y = 8$

For the following pairs of lines find the equation of the bisector of the angle in which the origin does not lie.

6. $4x + 3y = 1$
   $24x - 7y = 3$
7. $12x + 35y = 5$
   $8x - 15y = 2$
8. $8x - 6y = 7$
   $7x + 24y = 5$
9. $3x - 4y = 6$
   $4x + 3y = 2$
10. $1x - 2y = 3$
    $2x + 1y = 5$

2 -7 Slope Intercept Form

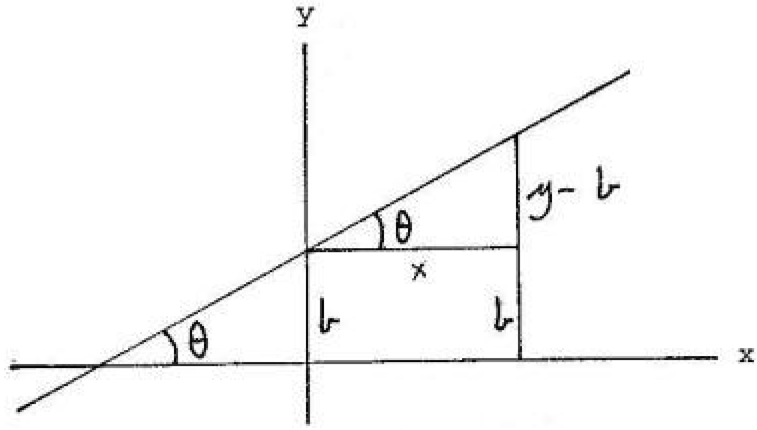

Fig. 2-20

See Fig. 2-20. Let a straight line cut off an intercept b on the y axis and make an angle θ with the x axis the tan of which we shall call m. Then

$$(y - b)/x = \tan \theta = m$$

(1) $$y = m x + b$$

Any equation may be put in the slope intercept form . For example the equation

$$a_1 x + a_2 y = a_3$$

may be written

$$y = - (a_1 /a_2) x + a_3 /a_2$$

Here the slope is $- (a_1/a_2)$ and the intercept a_3/ a_2. In Mutation Geometry the slope intercept form is not of too much importa nce but we have put it in for completeness.

Exercises

Find the slopes and intercepts of the following equations:

1 $2 x + 4 y = 5$ 2 $3 x - 5 y = 8$

2 - 8 Miscellani

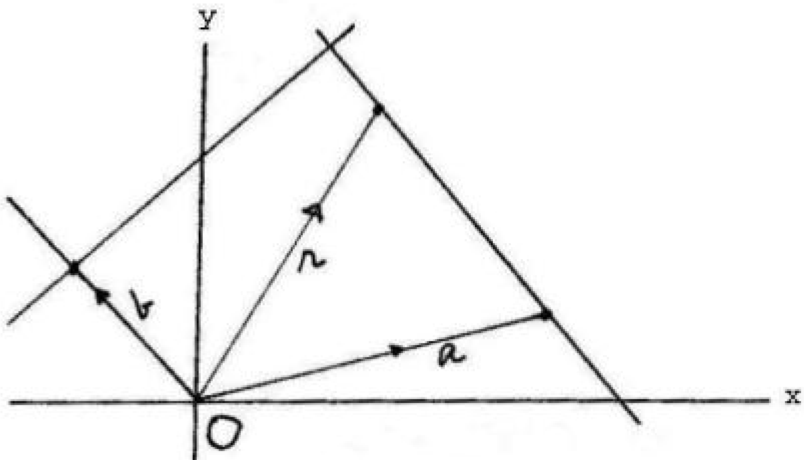

Fig. 2-21

Find the equation of a line passing thru a given point and perpendicular to a line whose equation is given. See Fig. 2-21 Let the given point be denoted by the vector a and let the equation of the given line be written

$$b \cdot r = c$$

If r is a vector to any point on the required line then r - a is perpendicular to $\breve{b}$ since b is perpendicular to the given line thus we may write

$$\breve{b} \cdot (r - a) = 0$$

(1) $$\breve{b} \cdot r = a \cdot \breve{b}$$

Equation (1) is the required equation. As a numerical example we shall find the equation of a line that passes thru the point a given by

$$a = a (2, 1) = 2 i + 1 j$$

and perpendicular to the line whose equation is

$$- 2 x + 1y = 4$$

$$a = 2i + 1j$$

$$b = -2i + 1j$$

$$\breve{b} = -1i - 2j$$

$$\breve{b} \cdot r = -x - 2y$$

$$\breve{b} \cdot a = -4$$

$$x + 2y = 4$$

is the required equation. On inspection it is seen to pass thru the given point and the cos of the angle between it and the given line is zero showing that it is perpendicular to the given line.

Now one would know that any line perpendicular to the given line could be written

$$x + 2y = k$$

and putting the coordinates of the given point a into this equation k equal to 4.

If it had been required to find the equation of a line that would pass thru, a given point and be parallel to a given line instead of perpendicular to it our equation would have been:

$$b \cdot r = a \cdot b$$

Putting in the values of a and b given above one gets:

$$-2x + y = -3$$

We now do a slightly more general problem along the same line: Find the equation of a line passing thru a given point and making a given angle with a line whose equation is given. We shall use the point and equation in the last illustration. Let the sign of the angle which the required line makes with the given line be S and the cos of the angle which the required line makes with the normal b be s. We may then write the alpha prototype equation:

$$b' \cdot (r - a)' = s$$

We solve this alpha prototype equation getting, according to (2) page 6 chapter 1, the following expression

$$(r - a)' = s\,b' \pm p\,\breve{b}'$$

$$p = \sqrt{1 - s^2}$$

If we multiply both sides of the second equation above by

$$(\pm p\,b' + s\,\breve{b}')$$

and at the same time multiply by the magnitude of each vector we get:

$$(\pm p\,b + s\,\breve{b}) \cdot (r - a) = 0$$

$$(2) \qquad (\pm p\,b + s\,\breve{b}) \cdot r = a \cdot (\pm p\,b + s\,\breve{b})$$

If the angle should be required to be 90 degrees then p is zero and s becomes one and equation (2) identifies itself with equation (1) which it should, granting that (1) was correct.

Equation (2) also gives a true answer when the line is required to be parallel to the given line. It includes all the special cases in its generality.

To solve this problem by the older conventional methods one can find the slope of the required line from the slope of the given line and the given angle and then one has a point and a slope with which to solve the problem.

Find the equation of a line which shall pass thru the point of intersection of two lines whose equations are given and will make a given angle with a third line whose equation is also given. Let the equations of the first two lines be

$$a \cdot r = h$$

$$b \cdot r = k$$

Let the third line with which the required line is to make a given angle be

$$c \cdot r = f$$

Let s, as before , be the sine of the angle which the required line makes with the third line.

One way to solve the problem is to solve the first two equations for their point of intersection and then (2) gives us the answer. We prefer to do it in a slightly different fashion. If three lines pass thru a common point in a plane one of them can be written as a linear combination of the other two. Thus we write our required equation as a linear combination of the first two equations:

$$U (a \cdot r - h) = b \cdot r - k$$

where U is some scalar . Rearranging we get

$$(U a - b) \cdot r = U h - k \qquad (3)$$

Now the expression $(U a - b)$ is normal to the required line and from the prototype solution the expression $(s c \pm p \breve{c})$ is along the required line so these two expressions are normal to each other therefore we can write:

$$(U a - b) \cdot (s c \pm p \breve{c}) = 0$$

$$U = \frac{b \cdot (s c \pm p \breve{c})}{a \cdot (s c \pm p \breve{c})}$$

We now put this value of U back into (3) and it becomes the known required solution. Note that there are two solutions in both equations (2) and (3). This is to be expected. This mode of solution seems slightly more elegant and one does not have to solve the two equations.

We shall offer some practice in numerical exercises at the end of the miscellani.

Exercises

1. Find the equations of the bisectors of the angles formed by the two lines whose equations are

$$x = 4$$

$$y = 2$$

2. Find the equation of a line passing thru the point (2, 3) and perpendicular to the line whose equation is

$$y = 3x - 4$$

3. Find the equation of a line passing thru the point of intersection of the first two of the three lines listed below and perpendicular to the third line:

$$x + y = 5$$

$$2x - y = 1$$

$$3x - 4y = 6$$

4. Find the equation of a line thru the point (2, 1) making an angle of 45 degrees with the line $y = 3x - 2$

5. Find the equation of a line thru (3, 2) and making an angle arc sin 3/5 with the line $y = 2x - 2$.

6. Find the equation of a line passing thru the intersection of the lines $x + y = 5$ and $2x - y = 1$ and making an angle arc sin 4/5 with the line $3x - 2y = 4$.

7. Find the equations of the lines thru (3, 4) and passing at a distance 1 from the point (5, 2).

8. Find the equations of the lines thru (2, 1) and passing at a distance 2 from the point (4, 0).

9. Determine x so that the three points (- 2, 1), (x, 4) and (3, 5) are collinear.

10. The slope of the hypotenuse of an isosceles right triangle is 2. what is the slope of each of its legs?

11. Find to the nearest degree the acute angle which the line thru the points (1, 2) and (3, 4) makes with the y axis.